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in Algebra

FOR FIFTH BOOK, PUBLIC SCHOOL, LEAVING, AND PRIMARY CLASSES

BY

G. E. HENDERSON

Editor of "The Canadian Teacher" and "The Entrance"

AND

E. W. BRUCE, B.A.

Principal Huron Street Public School, Toronto

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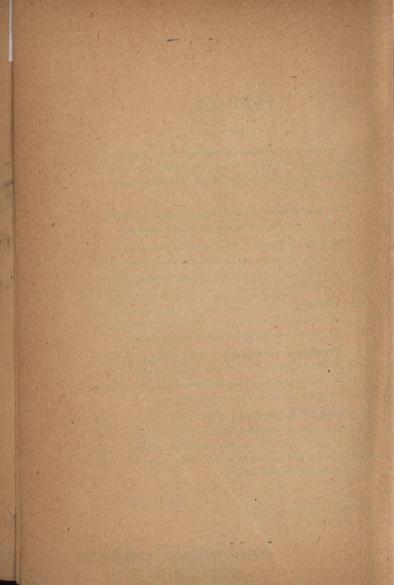

PREFACE.

The authors, who have prepared these exercises in Algebra at the request of many teachers, most respectfully request a consideration of the following points in connection with them :

I. No Answers. In the Pupils' edition no answers are provided. These are in the Teachers' edition.

II. Saving in Time. The time of the teachers is too valuable to be taken up in the dictation of problems to a class, when for a mere trifle each pupil may be provided with exercises.

III. Writing. The possession of these exercises by the pupils will tend to preserve their hand-writing by doing away with copying questions from dictation.

IV. Problems Grouped. The subjects are arranged in what appears to be the most natural order ; and, in accordance with the recommendation of teachers, the examples for exercises are very numerous.

V. Book of Exercises. This book is not in any sense designed to displace either the teacher or the authorized text-book. It merely furnishes ready to the teacher's hand many bright, crisp, new exercises with which to enforce his teaching.

THE AUTHORS.

Toronto, December, 1898.

EXERCISES IN ALGEBRA

For Fifth, Public School Leaving and Primary Classes.

Coefficient, Exponent, and Root, in Simple and Compound Expressions.

EXERCISE I.

If $a=5$, $b=3$, $c=2$, $x=4$, $y=1$, $z=0$, find the values of

A.

(1) $7a$. (2) $13b$. (3) $17c$. (4) $19x$. (5) $25y$.
(6) $15z$. (7) $5ab$. (8) $8bc$. (9) $9cx$. (10) $16xy$.
(11) $23xz$. (12) $37yz$. (13) $11xa$. (14) $14ax$. (15) abc.
(16) bcx. (17) $3xyz$. (18) $4axy$. (19) $7abx$. (20) $13bcx$.

B.

(1) a^2. (2) x^2 (3) y^2. (4) z^2. (5) $5a^3$.
(6) y^3. (7) $5a^3$. (8) $7c^3$. (9) $9x^3$. (10) $15y^3$.
(11) $18z^3$. (12) b^4. (13) a^2b^3 (14) b^3c^4. (15) x^2a^3.
(16) c^4y^5. (17) c^b . (18) a^x . (19) $5b^c$. (20) $12ax^b$.

C.

(1) $\frac{1}{2}ac$. (2) $\frac{2}{3}abc$. (3) $\frac{7}{9}b^2a$. (4) $\frac{3}{4}c^5$. (5) $\frac{9}{10}ax^2$.

(6) $\frac{11}{12}y^2z$. (7) 7^b . (8) 6^a . (9) 19^y (10) $\frac{8}{15}a^2b^3$.

5

(11) $\dfrac{x^3}{64}$.   (12) $\dfrac{b^4}{27}$.   (13) $\dfrac{abc}{19}$.   (14) $\dfrac{bca}{21}$.   (15) $\dfrac{xyz}{23}$.

(16) $\dfrac{1}{12}abx$ (17) $\dfrac{19}{20}x^2a^2$ (18) $\dfrac{3}{8}x^a$.   (19) $\dfrac{5}{6}b^a$ (20) $\dfrac{a^c\ b^x}{cx}$

D.

(1) $\sqrt{(5a)}$.   (2) $\sqrt{(27b)}$.   (3) $\sqrt{(2cx)}$.
(4) $\sqrt{(12xb)}$.   (5) $\sqrt{(15ab)}$.   (6) $\sqrt{(16xy)}$.

(7) $\sqrt{(2a^2c)}$.   (8) $\sqrt{\left(\dfrac{6bc}{4xy}\right)}$ (9) $\sqrt{\left(\dfrac{2b^2c^3}{5a^3x^2}\right)}$

(10) $\sqrt{\left(\dfrac{1}{6bcx}\right)}$ (11) $\sqrt[3]{(5a^2b^3)}$.   (12) $\sqrt[3]{(3b^2cx)}$.

(13) $\sqrt[3]{(5xyz)}$ (14) $\sqrt[3]{(2a^b\ c^2)}$ (15) $\sqrt[3]{\dfrac{1}{x^2c^2}}$

(16) $\sqrt[3]{\left(\dfrac{cx}{9a^3b}\right)}$ (17) $\sqrt[3]{\left(\dfrac{15a^2b^2}{36bc}\right)}$ (18) $\sqrt[3]{\left(\dfrac{25abc}{16bcx}\right)}$

EXERCISE II.

If $a=3$, $b=2$, $c=0$, $x=8$, $y=6$, $z=5$, find the values of :

A.

1. $12a+13b+14c+15x+16y+17z$.
2. $9z-8y+7x-6c+5b-4a$.
3. $ab+bc-cx+xy-yz$.
4. $14c-3a-5b+21x+17y-3z$.
5. $abc+bcx+cxy+xyz$.
6. $5az+6by-8cx+4bz-13byc$.
7. $a^2+b^2+c^2+x^2+y^2+z^2$.
8. $x^3+y^3+z^4-a^3-b^3-c^3$.
9. $a^2+b^2+c^2+2ab+2ac+2bc$.
10. $x^2+2xy+y^2-2xz-2yz+z^2$.
11. $a^3+3a^2x+3ax^2+x^3$.
12. $y^3-3y^2b+3yb^2-b^3$.

13. $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4.$
14. $81x^2 + 144xy + 64y^2 - 49z^2.$
15. $16x^2 + 40xy + 25y^2 - 9a^2 - 24ab - 16b^2.$

B.

1. $\dfrac{b}{a} + \dfrac{a}{x} + \dfrac{y}{z} + \dfrac{c}{y}$

2. $\dfrac{4a}{5b} + \dfrac{9b}{4x} + \dfrac{8b}{xy} - \dfrac{3a}{7y}$

3. $\dfrac{12a}{xb} - \dfrac{3z}{5x} - \dfrac{7ab}{xy} + \dfrac{3xy}{2ab}$

4. $\dfrac{2a + 5b}{x} + \dfrac{3b + 2y}{4z} - \dfrac{a + b + c}{x + y + z}$

5. $\dfrac{a + c}{a - c} + \dfrac{x + b}{x - b} + \dfrac{x + y}{x - y} + \dfrac{y + z}{y - z}$

6. $\dfrac{xy + 2xz + 3yz}{ab + 2ac + 3bc} - \dfrac{4ax + 3ac}{xy - xz + yz}.$

7. $\dfrac{a^2b^2 + 1}{a^2 + b^2} - \dfrac{1 + a^2c^2}{c^2 + a^2} + \dfrac{4x + y^2 + y^2z^2}{y^2}.$

8. $\dfrac{a^2 + b^2}{x} + \dfrac{b^2 + c^2}{y} + \dfrac{c^2 + a^2}{z}.$

9. $\dfrac{12a^3 - b^2}{3a^2} + \dfrac{2x^2}{x + y^2} - \dfrac{a + b^2 + c^3}{a + z}.$

10. $\dfrac{x^2}{y^2} + \dfrac{2bx}{ay} + \dfrac{b^2}{a^2} - \dfrac{2cx}{yz} - \dfrac{2bc}{az} + \dfrac{c^2}{z^2}.$

EXERCISE III.

If $x = 8$, $y = 6$, $z = 1$, $a = 9$, $b = 4$, find the values of

1. $\dfrac{5}{3}x - \dfrac{1}{9}y^3 + \dfrac{7}{8}b^2.$

2. $\dfrac{5}{27}ax - \dfrac{32}{b^2} - \dfrac{6x}{abz}.$

3. $\dfrac{3x^2y}{zab^2} - \dfrac{5b}{x}$.

4. $\sqrt[3]{\left(\dfrac{6zb^4}{a^2}\right)} + 2\sqrt{\left(\dfrac{3x^3}{4y^3}\right)}$

5. $\dfrac{2}{9}xy^2 - \sqrt{\left(\dfrac{a}{z^3}\right)} - yz^3b.$

6. $\sqrt[3]{(yab)} - \dfrac{1}{8}y^2 + \dfrac{8a^2}{by^2}.$

7. $\dfrac{3}{4}xz - \sqrt{\left(\dfrac{y^2}{9b}\right)} - \sqrt[3]{\left(\dfrac{by}{a^2}\right)}$

8. $\dfrac{5b^3y^2}{12ax^2} - \sqrt[3]{\left(\dfrac{a^4x}{b^2y^2}\right)} + \sqrt{\dfrac{xy^3}{3a}}$

9. $\sqrt{\left(\dfrac{2x}{9z}\right)} + \sqrt{\left(\dfrac{4ab}{3xy}\right)} + \sqrt{\left(\dfrac{abz}{12xy}\right)}$

10. $\sqrt{\left(\dfrac{x^2y}{9ab^2}\right)} + \sqrt[3]{\left(\dfrac{39a^2z}{208xy^2}\right)} + \sqrt[3]{\left(\dfrac{57ay^4}{38a^2b^3}\right)}$

ADDITION.

EXERCISE IV.

Find the sum of :

A.

1. $2a + 3b + 4c$; $5a + 6b + 7c$; $8a + 9b + 10c.$
2. $12x + 11y + 10z$; $9x + 8y + 7z$; $2x + 5y + 3z.$
3. $a + 3b - 4c$; $-3a + 3b + 3c$; $2a - 3b + c.$
4. $4a + 3b - 2c$; $-a + 3b + 2c$; $2a - b + 3c.$
5. $-4x + 3y + 2z$; $x - 2y + z$; $x + y - 3z.$
6. $-x + y + 4z$; $3x - 2y + 2z$; $2x - 3y - z.$
7. $-2a + 3b - 8c$; $a - b + c$; $4a + 3b - 5c.$
8. $-14a - 18b + 19c$; $13a + 15b + 8c$; $a + 6b + 8c.$
9. $26a - 16b - 3c$; $13a - 10b + 4c$; $a + 19b - 3c.$
10. $-17a - 11b + 6c$; $10a + 6b + 7c$; $6a + 5b - 2c.$

11. $4ax + 8by - 3cz$; $ax + 3by - cz$; $-3ax - 2by + 4cz$.
12. $20p + q - r$; $3p - 20q + r$; $p + q - 20r$.
13. $6m - 13n + 5p$; $8m + n - 9p$; $m + 2n + 5p$.
14. $5ab + 6cy - 3$; $-8ab - 3cy + 11$; $-7ab + 12cy + 7$.
15. $13xy - 5z + 8$; $-7xy + 5z - 18$; $-6xy + 13z + 10$.

B.

1. $-4ab + 6bc - 7ca$; $5ab - 3bc + 7ca$; $-2ab - 2bc + 4ca$.
2. $14ab - 28bc - 17ca$; $13ab + 45bc - 3ca$; $12ab - 18bc + 10ca$
3. $pq - qr - rp$; $-pq - qr + rp$; $pq - qr + rp$.
4. $x + y + z$; $3x - 4y + z$; $2x + 3y - 2z$.
5. $3a + 24b + 17c$; $16a - 23b - 9c$; $a - 3b + c$.
6. $4xy - 9yz + zx$; $23xy - 14yz + zx$; $12xy - 13yz - zx$.
7. $47x - y + 63z$; $-27x + 5y - 13z$; $-22x - 47y - 19z$.
8. $23a - 17bc - 2d$; $5a - 16bc - 13d$; $-a + 9bc - d$.
9. $ax - 4by + 3cz$; $13ax - 9by + 7cz$; $-5ax + 7by - 14cz$.
10. $2a + 3b + c$; $4b + 5c - 6d$; $17a - 13c + 19d$.

EXERCISE V.

Add together the following expressions :

A.

1. $3ab - 2bc - 3ca$; $2ab + 3ca + 6abc$; $-5ab + 2bc - 5abc$.
2. $3x^2 - 5xy + 6y^2$; $4x^2 + 6xy - 3y^2$; $4y^2 + 6x^2 - 7xy$.
3. $3a^2 - 7ab + 5b^2$; $4ab - 5a^2 + 3b^2$; $b^2 - 4a^2 + 9ab$.
4. $x^2 - xy + y^2$; $3x^2 + 4xy - 5y^2$; $6x^2 + xy + 5y^2$.
5. $x^2 + xy + y^2$; $-z^2 + yz - y^2$; $-x^2 + xz + z^2$.
6. $a^3 - a^2 + a - 1$; $2a^3 - 4a^2 - 6a + 7$; $5a^3 - 4a^2 - a + 9$.
7. $5x^4 - 3x^3 + 2x^2$; $x^4 + 7x^3 - 8x^2$; $9x^4 - 12x^3 - 15x^2$.
8. $12x^3 + 5x - 8$; $3x^3 - 4x^2 - 6$; $20x^2 - 41x - 19$.
9. $a^3 - ab - bc$; $ab + b^3 - ca$; $ca + bc + c^3$.
10. $5a^3 - 3c^3 + 2d^3$; $b^3 - 2a^3 + 2d^3$; $4c^3 - 2a^3 - 3d^3$.
11. $7x^3 - 8x + 5$; $2x^3 + x - 2$; $x^2 - 7x^3 - 3x + 9$.
12. $x^2 + y^2 + 2xy$; $2z^2 - 3y^2 - 4yz$; $2x^2 - 2z^2 - 3xz$.
13. $4x^3 + 5x^2y + 5xy^2$; $-3x^2y - 7xy^2 - 2x^3$; $7xy^2 - 8x^2y$.

14. $a^3 - 3a^2b + 8abc$; $a^2b - 10abc + c^3$; $b^3 + 2a^2b - abc$.

15. $20x^3 + 20x^2y - 3xy^2 + 14y^3$; $-17x^3 + 13x^2y - 10xy^2$
      $-5y^3$; $19x^3 + 18x^2y + 15xy^2 - 5y^3$; $-12x^3$
      $-13x^2y - 23xy^2 + 18y^3$.

B.

1. $\dfrac{1}{2}a - \dfrac{1}{3}b + \dfrac{1}{4}c$; $-a + \dfrac{3}{4}b - \dfrac{5}{8}c$; $\dfrac{3}{4}a - b - c$.

2. $2a + \dfrac{7}{8}b - \dfrac{11}{12}c$; $\dfrac{9}{10}a - \dfrac{5}{12}b - \dfrac{5}{8}c$; $\dfrac{7}{8}a + \dfrac{13}{15}b - \dfrac{11}{16}c$.

3. $\dfrac{1}{2}x^2 - \dfrac{1}{3}xy + \dfrac{3}{4}y^2$; $-\dfrac{1}{3}x^2 + \dfrac{1}{4}xy - \dfrac{1}{5}y^2$; $\dfrac{5}{2}x^2$
      $-\dfrac{7}{3}xy - \dfrac{7}{8}y^2$.

4. $\dfrac{a}{2} - \dfrac{b}{3} + \dfrac{c}{4}$; $\dfrac{b}{2} - \dfrac{c}{3} + \dfrac{a}{4}$; $\dfrac{c}{2} - \dfrac{a}{3} + \dfrac{b}{4}$.

5. $-2a + \dfrac{5}{2}c$; $\dfrac{8}{3}b - 3c$; $-\dfrac{1}{3}a - 2b$.

6. $\dfrac{7}{8}x^2 - \dfrac{2}{3}xy + \dfrac{7}{10}y^2$; $-\dfrac{5}{6}x^2 + \dfrac{14}{15}xy - y^2$; $\dfrac{1}{2}x^2 - xy$
      $+\dfrac{2}{5}y^2$.

7. $-\dfrac{3}{4}x^3 + 4ax^2 - \dfrac{5}{8}a^2x$; $x^3 - \dfrac{37}{8}ax^2 + \dfrac{1}{2}a^2x$; $-\dfrac{1}{2}x^3$
      $+\dfrac{3}{4}a^2x$.

8. $\dfrac{1}{2}a^3 - 2a^2b - \dfrac{3}{2}b^3$; $\dfrac{3}{2}a^2b - \dfrac{3}{4}ab^2 + 2b^3$; $-\dfrac{3}{2}a^3$
      $+ab^2 + \dfrac{1}{2}b^3$.

9. $\dfrac{17}{8}m - \dfrac{15}{16}n + p$; $\dfrac{5}{12}m + \dfrac{7}{8}n - \dfrac{23}{27}p$; $\dfrac{11}{16}m + n$.

10. $a^3 - \dfrac{5}{4}b^3 - 2abc$; $b^3 + \dfrac{3}{2}c^3 - \dfrac{1}{3}abc$; $-\dfrac{5}{2}a^3$
$-\dfrac{11}{12}c^3 - \dfrac{9}{5}abc$.

SUBTRACTION.

EXERCISE VI.

Subtract:

A.

1. $2x + 3y + 5z$ from $7x + 8y + 9z$.
2. $4a + 7b + 9c$ from $12a + 15b + 11c$.
3. $2a - 3b - 4c$ from $5a + 7b + 6c$.
4. $4x - 5y - 6z$ from $6x - 8y - 9z$.
5. $3a - 4b + c$ from $4a - 3b - 4c$.
6. $2x - 7y + z$ from $9x + 10y - 16z$.
7. $14a - 29b + 8c$ from $10a + 4b + 5c$.
8. $-9x - 12y + 13z$ from $x + y - z$.
9. $-4a + 3b - 4c$ from $2a - b + c$.
10. $-13x - 14y + 15z$ from $-9x + 13y - 15z$.
11. $3ab + 4bc - 6cd$ from $5ab - 2bc + cd$.
12. $ab - cd + ac - bd$ from $-ab + cd - ac + bd$.

B.

1. $2ab + 4cd + 5ac - 7bd$ from $2ab + 5cd - 3ac - bd$.
2. $xy - yz + zx$ from $-xy + yz - zx$.
3. $9p - 14q + 3r$ from $5q - 3p + 2r$.
4. $8a - 3b + 7c$ from $5c + 2a - 5b$.
5. $8 - c + b - a$ from $a + b + c + 3$.
6. $2x - 2y - 3z$ from $x + y$.
7. $2a + 4c$ from $a - b - c$.
8. $5ab - 17xy + 18$ from $9ab + 3xy - 23$.

9. $-5a+11xy-19$ from $2a+8-2xy$.
10. $-\frac{3}{4}x+\frac{2}{3}y-\frac{9}{7}z$ from $\frac{5}{4}x-\frac{4}{3}y+\frac{5}{7}z$.
11. $\frac{1}{4}x+\frac{5}{6}y+2$ from $\frac{1}{3}x-\frac{1}{4}y-1\frac{1}{2}$.
12. $\frac{2}{3}a-\frac{3}{5}b-\frac{1}{2}c$ from $-a-5b$.

EXERCISE VII.

From :

A.

1. $4xy-7yz+9zx$ take $-2xy+3yz-12zx$.
2. x^3+12x^2+5x-2 take $3x^3-2x^2-7x-5$.
3. $-5x^2y^2-15xy^3+8y^4$ take $4x^2y^2+7xy^3-17y^4$.
4. $-10+4ab-6a^2b^2$ take $5-3ab+2a^2b^2$.
5. $a^2bc+b^2ca+c^2ab$ take $4a^2bc-5b^2ca+6c^2ab$.
6. $-5a^2b+8ab^2-20$ take $-3a^2b-8ab^2+17$.
7. $4x^2y-3xy^2+21$ take $5x^2y-3xy^2-20$.
8. $-a^2-b^2-c^2-d^2$ take $a^2-c^2+d^2-b^2$.
9. $4x^3y-2x^2y^2+xy^3$ take $3xy^3+4x^2y^2-5x^3y$.
10. $a^3-3a^2b+3ab^2-b^3$ take $-a^3+3a^2b-3ab^2+b^3$.
11. $18x^3-16x^2+14x-12$ take $10-12x+14x^2-16x^3$.
12. $-3a^2+4ab-5b^2$ take $-7a^2+3b^2-2c^2$.

B.

1. a^3+b^3-3abc take b^3-c^3-2abc.
2. $5x^4-7x^3-3x^2+9$ take $x^4+6+2x-4x^3$.
3. $-3a^3-5b^3+c^3+5abc$ take $a^3+b^3+c^3-3abc$.
4. $x^4-x-1+x^2$ take $1+x-x^5-x^4-x^3$.
5. $7ab^2+16a^2b+b^3$ take $a^3+b^3+7a^2b-8ab^2$.
6. $x^4-3x^3-10x+5$ take $3x^4-2x^2+7x-9$.
7. $\frac{1}{2}a+b$ take $a-\frac{1}{3}b$.
8. $\frac{3}{4}x^2-\frac{7}{12}x-1\frac{1}{16}$ take $-\frac{5}{8}x^2+\frac{5}{8}x-1\frac{5}{16}$.
9. $\frac{2}{3}a^2-\frac{5}{6}a-15$ take $-\frac{2}{9}a^2-a-\frac{3}{4}$.
10. $\frac{3}{5}x^2-\frac{3}{4}x+\frac{3}{7}$ take $\frac{1}{3}+\frac{1}{4}x-\frac{1}{6}x^2$.
11. $\frac{5}{9}x^2-\frac{4}{3}xy^2-y^2$ take $\frac{1}{3}x^2y-\frac{5}{7}y^2$ $\frac{3}{4}xy^2$.
12. $\frac{1}{9}a^2-2ax^2-\frac{2}{5}ax^3$ take $\frac{1}{6}ax^2+\frac{1}{4}a^2-\frac{3}{5}ax^3$.

EXERCISE VIII.

1. Add $4x + 7y + 13z$ five times in succession to $x - 41y - 72z$.

2. To the sum of $3a - 4b - 17c$ and $4c - 5b - a$ add the sum of $a - 5b - 19c$ and $a + c + 15b$.

3. From $3x^3 - 2x^2 - 5x + 7$ take the sum of $8x^3 - 5x^2 + 7x - 2$ and $8x^2 - 9x - 19$.

4. Subtract $5 + a - 9a^2 + 7a^3$ from the sum of $8 - 2a - 13a^2$ and $6a - 19a^2 - 27a^3$.

5. Find the sum of $19a - 27b - 36c$ and $-28a + 27b - 39c$, and subtract the result from $2a - 3b - 5c$.

6. Take $x^2 - 3y^2$ from $5xy + 7y^2$, and add the remainder to the sum of $5x^2 - 9xy + 3y^2$ and $-8xy - 11y^2$.

7. Add together $5x^3 + 7x^2y - 9xy^2 + 18y^3$ and $-2x^3 - 5xy^2 - 7x^2y - y^3$, and diminish the result by $-x^3 - x^2y - xy^2 - y^3$.

8. Take $14a^2 - 14a + 3$ from unity, and add $5 + 13a - 9a^2$ to the difference.

9. What expression must be subtracted from $19x^2 - 3x + 4y - 7$ to leave $x^2 - y - 9$?

10. What expression must be added to $5ab - 11ac + 12bc$ to produce $ab + 5bc - 6ac$?

11. To what expression must $8x^2 - 9x + 5$ be added to produce zero?

12. Subtract $5x^3 + 4x^2 - 5x - 9$ five times in succession from $x^3 + 13x - 18$.

13. From $5x^2 + 6xy - 5y^2 - 12xz - 3yz - 8z^2$ take $2x^2 - 3y^2 + 4xz - 5z^2 + 6yz - 7xy$.

14. From $a^5 - 4a^3b^2 - 8a^2b^3 - 17ab^4 - 12b^5$ take in succession $a^5 - 2a^4b - 3a^3b^2$; $2a^4b - 4a^3b^2 - 6a^2b^3$; $3a^3b^2 - 6a^2b^3 - 9ab^4$; and $4a^2b^3 - 8ab^4 - 12b^5$.

15. By how much does $2x - 3$ exceed $5x - 17$?

16. Subtract the sum of $15l - 9m + 3n - p$ and $4m - 5n + p + l$ from $13l - 11m - 9n$.

MULTIPLICATION.

EXERCISE IX.

Multiply together :

A.

1. $5x^3$ and $4x^2$.
2. $4a^4$ and $5a^5$.
3. $3xy$ and $7xy$.
4. $4ab$ and ab.
5. $3x$ and $8y$.
6. $17a$ and $2b$.
7. $5a^7$ and a.
8. $7x$ and $9x^6$.
9. $3a^2b$ and $10ab^2$.
10. $9xy^2$ and $8x^3$.
11. $11a^3b^3$ and $11a^3b^3$.
12. $4a^2b^3$ and $8a^5$.
13. x^3y^3 and $6a^2x^4$.
14. $7a^4b^7c^3$ and $5a^2b^6c$.
15. xyz and abc.

16. $9a^2x$ and $8cx$.
17. $3a^4bx$ and $7b^3x^2$.
18. $5x^3y^2$ and $6a^3x^3$.
19. $2a^2b$ and a^5b^7.
20. $7x^5y^7$ and $4x^2yz^3$.
21. $4a^3bx$ and $12a^2b^2c$.
22. $19x^3yz^2$ and $4x^2y^4z^3$.
23. $7abc$ and $14\,xyz$.
24. $a^7b^6c^5$ and $7a^5bc$.
25. ay^2z and $bxyz^3$.
26. $9m^2n^3p^4$ and mn^5p^3.
27. $8c^4x^3$ and $7a^5c^4$.
28. acx and cxy.
29. $5a^2b^2c^2$ and bcx^2.
30. $14a^7$ and $16a^{17}b^{15}m^2$.

B.

1. $ab + bc$ and a^2b.
2. $x^2y + xy^2$ and $5x^3y^4$.
3. $8x + 3y$ and $7x^2$.
4. $5ab - 9bx$ and $3a^2b^3x^4$.
5. $4a^2 - 3b$ and $5ab$.
6. $8x^2 - 9xy$ and $7x^2$.
7. ax^2 and $-3ax$.

8. $-3abx$ and $-9abx$.
9. $-ab^2c^2d$ and $-3a^2bc^2d^4$.
10. xyz and $-8x^2y^3z^4$.
11. $-bc$ and a^2bc^3.
12. $-7x^3y$ and $-9xy^3z^4$.
13. $6m^2n^3p$ and mnp^2.
14. $-a^2bc$ and -9.

15. $a^2 + b^2 - c^2$ and abc.
16. $ab - bc - ca$ and $a^2b^3c^4$.
17. $5a^2 - 9b^2 - 11c^2$ and $-3ac$.
18. $9x^2 + 8xy^2 - 7y^3$ and $14\,x^3$.
19. $-3x^3y - 5xy^3$ and $-7x^3y^3$.
20. $-8x^2yz + 3xy^2z - 13xyz^2$ and $-xyz$.
21. $-a^2bc - b^2ca - c^2ab$ and $-ab$.

22. $x^2y^3 - y^3z^4 + z^4x^2$ and $5x^2y^2z^2$.
23. $2xy^2z^3 - 3x^2y^3z - 5x^3yz^2$ and $4xy^2z$.
24. $5x^3 - 4x^2 + 8x - 9$ and -13.
25. $-5a^3 - 7a^2 + 6a - 11$ and $10ab$.

EXERCISE X.

Find the product of:

A.

1. $4x^2 - 7x + 6$ and $2x - 3$.
2. $5x^3 - 6x^2 + 7x - 8$ and $4x + 5$.
3. $4a^4 - 3a^3 - 2a^2 - a - 8$ and $3a - 7$.
4. $2x^3 + 4x^2 + 8x + 16$ and $3x - 6$.
5. $x^3 + x^2 + x - 1$ and $x - 1$.
6. $7 - a + 5a^2 - 9a^3$ and $5 - 6a$.
7. $13 + x - 3x^2 - 7x^3$ and $2 - 9x$.
8. $5x^2 - 4xy + 7y^2$ and $3x - 4y$.
9. $7a^3 - 9a^2b + 10ab^2 + 6b^3$ and $7a - 4b$.
10. $x^3 - 2x^2a + 2xa^2 - a^3$ and $x + a$.
11. $16a^2 + 12ab + 9b^2$ and $4a - 3b$.
12. $8x^2 - 7x + 9$ and $5x^2 + 3x - 8$.
13. $a^2 - 2ax + 4x^2$ and $a^2 + 2ax + 4x^2$.
14. $10a^2 - 3ax + x^2$ and $10a^2 + 3ax - x^2$.
15. $a^3 - 3a^2b + 3ab^2 - b^3$ and $a^2 - 2ab + 3b^2$.
16. $x^3 - 6ax^2 + 18a^2x - 27a^3$ and $x^3 + 6ax^2 + 18a^2x + 27a^3$.
17. $3x^3 + 4x^2 + 5x + 6$ and $3x^3 - 4x^2 + 5x - 6$.

B.

1. $a + b + c$ and $a + b - c$.
2. $2a + 3b - 4c$ and $2a + 3b + 4c$.
3. $x^2 + xy + y^2$ and $x^2 - xy + y^2$.
4. $3x^2 - 2xy + 4y^2$ and $3x^2 + 2xy - 4y^2$.
5. $x^2 + 2xy + y^2$ and $x^2 - 2xy + y^2$.
6. $ab + cd + ac + bd$ and $ab + cd - ac - bd$.
7. $a^3 + 2a^2 + 2a + 1$ and $a^3 - a^2 + 2a - 1$.
8. $a^4 - 2a^3b + 3a^2b^2 - 2ab^3 + b^4$ and $a^2 + 2ab + b^2$.
9. $x^2 + y^2 + z^2 - xy - xz - yz$ and $x + y + z$.
10. $x^2 - xy + x + y^2 + y + 1$ and $x + y - 1$.

11. $27a^3 - 36a^2x + 48ax^2 - 64x^3$ and $3a + 4x$.
12. $x^{12} - x^9y^2 + x^6y^4 - x^3y^6 + y^8$ and $x^3 + y^2$.
13. $a^4 + 2a^3 + 3a^2 + 2a + 1$ and $a^2 - 2a + 1$.
14. $x^3 - ax^2 + a^2x - a^3$ and $x + a$.
15. $x^4 - 2x^3y + 3x^2y^2 - 2xy^3 + y^4$ and $x^2 + 2xy + y^2$.
16. $x^2 + ax + a^2$, $x^2 - ax + a^2$ and $x^4 - a^2x^2 + a^4$.

DIVISION.

EXERCISE XI.

Divide :

A.

1. x^5 by x^2.
2. x^{10} by x^3.
3. $3a^4$ by a^2.
4. $27a^4$ by $3a^3$.
5. $35x^6$ by $7x^3$.
6. $81a^5$ by $-9a^2$.
7. $40x^{11}$ by $-5x^8$.
8. x^3y^3 by x^2y.
9. a^7x^4 by a^2x.
10. $4a^2b^2c^5$ by ab^2c^3.

11. $15a^6b^6c^6$ by $-3a^4b^2c$.
12. $-20x^5y^3$ by $-4xy^2$.
13. $-48a^9$ by $-8a^2$.
14. $-35a^{10}$ by $-5a^5$.
15. $63a^7b^8c^9$ by $9a^5b^6c^7$.
16. $7a^2bc$ by $-7a^2bc$.
17. $-50y^3x^4$ by $-5yx^3$.
18. $18b^2yx^2$ by $-3xy$.
19. $-24a^4b^5c^6$ by $-3a^3b^4$.
20. $201a^3b^3x^3$ by -3.

B.

1. $4x^5 + 6x^4 + 8x^2 + 10x$ by $2x$.
2. $3a^4 - 15a^3 - 18a^2$ by $-3a^2$.
3. $15a^3b^3 - 6a^2b^2 + 12ab$ by $-3ab$.
4. $36a^3b^3c^2 - 48a^2b^3c^3 + 60a^3b^2c^3$ by $4abc^2$.
5. $a^2 - ab + ac$ by $-a$.
6. $a^4 - a^3b + a^2b^2 - ab^3$ by a.
7. $34x^4y^3 - 51x^3y^2 + 68x^2y$ by $-17x^2y$.
8. $72x^5y^6 - 45x^4y^3 - 18x^2y^2$ by $9xy^2$.
9. $81m^8n^7 - 84m^5n^6 + 27m^3n^4p$ by $3m^2n^2$.
10. $169a^4b - 117a^3b^2 - 91a^2b$ by $13a^2$.
11. $361b^5c^4 + 228b^4c^5 - 133b^3c^5$ by $19b^3c^2$.
12. $-144x^4 + 108x^3y - 96x^2y^2 + 60xy^3$ by $-12x$.

EXERCISE XII.

Divide :

A.

1. $x^2 + 16x + 60$ by $x + 10$.
2. $x^2 - 17x + 70$ by $x - 7$.
3. $a^2 - 11a + 30$ by $a - 6$.
4. $x^2 - 49x + 600$ by $x - 25$.
5. $3a^2 + 10a + 3$ by $a + 3$.
6. $5a^2 + 16a + 3$ by $a + 3$.
7. $4x^2 + 23x + 15$ by $4x + 3$.
8. $4a^2 + a - 14$ by $a + 2$.
9. $6a^2 - 31a + 35$ by $2a - 7$.
10. $15x^2 + 17xa - 4a^2$ by $3x + 4a$.
11. $9a^2 + 6ac - 35c^2$ by $3a - 5c$.
12. $60x^2 - 4xy - 45y^2$ by $6x + 5y$.
13. $96x^2 - 4xy - 15y^2$ by $12x - 5y$.
14. $9a^3 + 3a^2 + a - 1$ by $3a - 1$.
15. $7x^3 - 24x^2 + 58x - 21$ by $7x - 3$.
16. $x^3 + 2x^2 + 2x + 1$ by $x + 1$.
17. $x^3 + 4x^2y + 3xy^2 + 12y^3$ by $x + 4y$.
18. $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$ by $a + b$.
19. $a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$ by $a - b$.
20. $x^4 - 4x^3 + 6x^2 - 4x + 1$ by $x^2 - 2x + 1$.

B.

1. $x^4 - 5x^3 + 11x^2 - 12x + 6$ by $x^2 - 3x + 3$.
2. $a^4 + a^3 - 9a^2 - 16a - 4$ by $a^2 + 4a + 4$.
3. $x^4 + 10x^3 + 35x^2 + 50x + 24$ by $x^2 + 5x + 4$.
4. $18x^4 - 45x^3 + 82x^2 - 67x + 40$ by $3a^2 - 4x + 5$.
5. $14a^4 + 45a^3b + 78a^2b^2 + 45ab^3 + 14b^4$ by $2a^2 + 5ab + 7b^2$.
6. $x^5 - 2x^4 - 4x^3 + 19x^2 - 31x + 15$ by $x^3 - 7x + 5$.
7. $x^4 + 8x^3 - 4x^2 - 128x - 192$ by $x^2 - 16$.
8. $m^5 + 2m^4 + 4m^3 + 9m^2 - 31m + 15$ by $m^2 + 2m - 3$.
9. $a^3 + 3a^2b + 3ab^2 + b^3 - 1$ by $a + b - 1$.
10. $b^6 + 6b^5c + 15b^4c^2 + 20b^3c^3 + 15b^2c^4 + 6bc^5 + c^6$ by $b + c$.

11. $3a^2 + 8ab + 4b^2 + 10ac + 8bc + 3c^2$ by $a + 2b + 3c$.
12. $x^5 + y^5$ by $x + y$.   16. $81x^4 - 16y^4$ by $3x + 2y$
13. $a^5 - b^5$ by $a - b$.   17. $16a^4 - 81b^4$ by $2a - 3b$.
14. $x^6 - y^6$ by $x - y$.   18. $x^4 - 81y^4$ by $x - 3y$.
15. $a^4 - 16b^4$ by $a + 2b$.   19. $x^7 + 1$ by $x + 1$.
20. $x^6 - y^6$ by $x^3 - 2x^2y + 2xy^2 - y^5$.
21. $a^{12} + a^6 - 2$ by $a^4 + a^2 + 1$.
22. $x^3 + 3x^2y + 3xy^2 + y^3 + z^3$ by $x + y + z$.
23. $a^2 + 2ab + b^2 - c^2$ by $a + b - c$.
24. $a^2 - b^2 - c^2 + d^2 - 2ad + 2bc$ by $a - b + c - d$.
25. $x^3 + y^3 + z^3 - 3xyz$ by $x + y + z$.

BRACKETS.

EXERCISE XIII.

Simplify by removing brackets;

A.

1. $3a - b - (2a - b)$.
2. $a - (b - c) + a + (b - c) + b - (c + a)$.
3. $a + b + (7a - b) - (2a - 3b) - (5a + 4b)$.
4. $(a + x) - (b - x) - (a - b)$.
5. $1 - (1 - a) + (1 - a + a^2) - (1 - a + a^2 - a^3)$.
6. $(a - b + c) - (b + c - a) + (c - a + b) - (a - c + b)$.
7. $\{ a - (b - c) \} + \{ b - (c - a) \} - \{ c - (a - b) \}$.
8. $a - [2a - \{ 3b - (4c - 2a) \}]$.
9. $7a - [3a - \{ 4a - (5a - 2a) \}]$.
10. $6a - [4b - \{ 4a - (6a - \underline{4b}) \}]$.
11. $2x - [2x - \{ 2x - (2x - 2x - x) \}]$.
12. $3x - [5y - \{ 6z - (4x - 3y) \}]$.
13. $-[5a - (11b - 3a)] - [5b - (3a - 6b)]$. $\overline{}$
14. $a - 2b - [4a - 6b - \{ 3a - c + (5a - 2b - \overline{3a - c - 2b}) \}]$
15. $8x - \{ 16y - [3x - (12y - x) - 8y] + x \}$.
16. $3b - \{ 5a - [6a + 2(10a - b)] \}$.
17. $2x - (3y - 4z) - \{ 2x - (3y + 4z) \} - \{ 3y - (4z + 2x) \}$.

18. $3a^2 - [6a^2 - \{ 8b^2 - (9c^2 - 2a^2) \}]$.

19. $-4(a+d) + 24(b - c) - 2[c+d + a - 3\{ d + a - 4 (b+c) \}]$.

20. $[a - 5b - \{ a - (5c - \overline{2c - b} - 4b) + 2a - (a - \overline{2b + c} \}]$.

Without removing the brackets, divide :

21. $x^3 + (a+b+c)x^2 + (bc+ca+ab)x + abc$ by $x^2 + (a+b)x + ab$.

22. $x^4 - (5+a)x^3 + (4+5a+b)x^2 - (4a+5b)x + 4b$ by $x^2 - 5x + 4$.

23. $x^3 - (a-b)x^2 - (ab+2b^2)x + 2ab^2$ by $x - b$.

24. $ax^4 - (ap-b)x^3 + (aq-bp-c)x^2 + (bq+cp)x - cq$ by $ax^2 + bx - c$.

25. $2x^3 + (2a+3c)x^2 + (3ac-4b)x - 6bc$ by $2x + 3c$.

26. $x^4 + (a+b)x^3 + (a^2+ab+b^2)x^2 + (a^3+b^3)x + a^2b^2$ by $x^2 + ax + b^2$.

27. $x^4 - (a+c)x^3 + (b+ac)x^2 - bcx$ by $x - c$.

28. $x^6 + px^5 + (p-q+r)x^4 + (p^2-pq)x^3 + (pq+pr-qr)x^2 + p^2qx + pqr$ by $x^2 + px + r$.

29. $apx^4 + x^3(aq + bp) + x^2(ar+bq+pc) + x(qc+br) + cr$ by $ax^2 + bx + c$.

30. $x^4 + 2ax^3 - (n^2-1)a^2x^2 + 2na^3x - a^4$ by $x^2 - (n-1)ax + a^2$.

B.

Insertion of brackets.

1. Write the expression $ax - bx + cx - ay + by - cy$; first, with the x terms and the y terms bracketed respectively ; secondly, with the a terms, the b terms, and the c terms bracketed respectively.

2. In the expression $ax^3 - cx + 7 - dx^2 + bx - c - dx^3 + bx^2 - 2x$ bracket together the powers of x so as to have the sign $+$ before each bracket.

3. In the expression $-a^2x - 7a + a^2y + 3 - 2x - ab$ bracket together the powers of a so as to have the sign $-$ before each bracket.

In the following expressions bracket the powers of x so that the signs before all the brackets shall be positive :

4. $ax^4 + bx^2 + 7 + 3bx - 8x^2 - 3x^4 - 9x$.

5. $4bx^2 - 8 - 5x + ab + 6ax^3 + cx - 2x^2 - bx^3$.

6. $4 - 8x^3 + 6ax^2 - 3cx + 10ax^3 + 9x - 12x^2$.

7. $3cx^5 - 4abc + 5dx - 4bx^4 - 2a^2x^5 + 3x^4$.

In the following expressions bracket the powers of x so that the signs before all the brackets shall be negative:

8. $ax^2 + 4x - 2a^2x^4 - 3bx^3 - bx^4 - 3x^2$.

9. $7x^5 + 8x^3 - 3c^2x - abx^5 + 9ax - abcx^3$.

10. $ax^2 + a^2x^3 - 5x^2 - cx^3 - bx^2$.

11. $6b^2x^4 - 2bx - 3ax^4 - cx^4 - 5c^2x - 7x^4$.

12. $5ax^3 - 7bx + 7cx^2 - 6bx^2 + 3ax^2 + 2ax + 4cx^3$.

Express, by brackets, taking the terms (a) two, (b) three, together :

13. $2a - 3b - 4c + 5d - 4e + 3f$.

14. $-b - 5c + 6d - 3e + 4f + g$.

15. $-3x + 4y - 2z + 3a + 2b - c$.

16. $4c - 2d + 3e + 2x - y - 5z$.

17. $-2m + 3n + 4a - 6b - 5x + 7y$.

18. $3p + 2q - 4r - 5m + 3n - 2a$.

SIMPLE EQUATIONS.

EXERCISE XIV.

Solve the following:

A.

1. $7x + 5 = 5x + 11$

2. $12x + 7 = 8x + 15$

3. $16x - 11 = 7x + 70$.

4. $24x - 49 = 19x - 14$.

5. $3x + 23 = 78 - 2x$

6. $5x - 7 = 3x + 7$

7. $12x - 9 = 8x - 1$.

8. $124x + 19 = 112x + 43$.

9. $18 - 2x = 27 - 5x$.

10. $12x - 145 = 7x - 125$.

11. $26 - 8x = 80 - 14x$.

12. $266 - 6x = 2x - 186$.

13. $39 - 3x = 15x - 9$.

14. $75 - 25x = 30 - 20x$.

15. $6x - 22 = 14x + 12$.

16. $24 + 12x = 36 - 48$.

17. $16x = 38 - 12 + 3x$.

18. $5x - 35 + 63 = 9x$.

19. $2x + 3 = 16 - 2x + 3$.

20. $63(5 - x) = 72(x - 5)$.

21. $27x - 9(5x - 6) + 90 = 0$.
22. $4x - (6x - 35) = 5x - (3x - 7)$.
23. $24x - 5(18x + 6) + 12(7 - 8x) + 1566 = 0$.
24. $2x - 1 - 2(3x - 2) + 3(4x - 3) - 4(5x - 4) = 0$.
25. $13x - 21(x - 3) = 10 - 21(3 - x)$.

B.

1. $6(169 - x) - 2(78 + x) = 58x$.
2. $14x - 78 - 20x + 30 = 200 - 66x + 52$.
3. $163 - 15(2x - 5) = 157 - 21(x + 3)$.
4. $(x + 12)(x - 8) = (x - 6)(x + 1)$.
5. $(x + 7)(x - 3) = (x - 5)(x - 15)$.
6. $(2x - 7)(x + 5) = (9 - 2x)(4 - x) + 229$.
7. $(x + 5)(x - 9) + (x + 10)(x - 8) = (2x + 3)(x - 7) - 113$.
8. $5x - (3x - 7) - \{ 4 - 2x - (6x - 3) \} = 10$.
9. $14x - (5x - 9) - \{ 4 - 3x - (2x - 3) \} = 30$.
10. $25x - 19 - [3 - \{ 4x - 5 \}] = 3x - (6x - 5)$.
11. $(5 - 3x)(7 - 2x) = (11 - 6x)(3 - x)$.
12. $(3x^2 - 2x + 1)(2x - 1) = 6x^3 - 7x^2 + 9x - 6$.
13. $7x - \{ (3x - 2) - (5x + 3) \} = 27 - 2[3x - (x + 2)]$.
14. $5(5x - 4) - 4(4x - 3) + 3(3x - 2) - 2(2x - 1) = 2$.
15. $1.2x - .05 = .07x + .3x + 16.55$.
16. $2.1x + 5.25 - 4.4x + 5.5 = 3.08x - 5.39$.
17. $.375x - 1.875 = .12x + 1.185$.
18. $.72x - 1.08 + .45x + 4.05 = 2.8x - 28$.
19. $(x + 1)(x + 2)(x + 3) - (x + 4)(x - 3)(x + 5) = 84$.
20. $x^3 + 9x^2 + 4(7x - 1) = (x + 6)(x + 2)(x + 1)$.
21. $7x - \{ (3x - 2) - (5x + 3) \} = 27 - 2[3x - (x + 2)]$.
22. $(5x^2 - 7x + 4)(2x - 9) = 10x^3 - 59x^2 + 37x + 100$.
23. $(x + 3)(3x - 2) - (x + 5)(3x - 7) = 3x + 1$.
24. $(x + 1)(x + 3)(x + 5) = (x + 7)(x + 9)(x - 7)$.
25. $.6x + .75x - .16 = x - .583x + 5$.

EXERCISE XV.

SYMBOLICAL EXPRESSION.

A.

1. If **x** denotes a certain number, express :—double the number ; three times the number ; 5a times the num-

ber ; the square of the number ; the cube of the number ;
the square root of the number ; the cube root of the
number ; the sum of a times the number and b times the
number ; the difference between c times the number and
d times the number ; a number that is 8 less than 9 times
the number ; a number that is 13 greater than twice the
number.

2.   If a and b denote two numbers, of which a is the
greater, express :—the sum of two numbers ; the differ-
ence between two numbers ; the sum of the squares of two
numbers ; the difference between the squares of two
numbers ; the square of the sum of two numbers ;
the square of the difference of two numbers; six
times the product of two numbers ; the sum of the
squares of two numbers divided by the difference of the
numbers ; the sum of two numbers divided by the differ-
ence of the squares of the numbers ; the sum of the
cubes of two numbers divided by the sum of the numbers ;
the square root of the sum of the squares of two
numbers.

3.   If n be a whole number, express :— an even
number ; an odd number ; four consecutive numbers of
which n is the least ; four consecutive numbers of which
n is the greatest ; five consecutive numbers of which n
the middle one ; five consecutive even numbers of which
2n is the middle one ; five consecutive odd numbers of
which 2n + 1 is the middle one.

4.   If a and b are two digits, express :—the product
of the two digits in as many ways as you can ; the two
numbers of which these are the digits ; the product of
these two numbers ; the sum of these two numbers.

5.   If x, y and z are three digits, express :—the pro-
duct of these three digits in as many ways as you can ;
the six numbers of three digits each that can be formed
by them.

6.   Write algebraically all the numbers that can be
expressed by the three digits l, m, n.

7. A man makes a journey of x miles. He travels a miles by coach, b by train, and finishes the journey by boat. How far does he travel by boat?

8. If a man was x years old y years ago, how old will he be in z years?

9. How old is a man who x years ago was n times as old as his child then aged a years?

10. A man has a journey of m miles; he travels a hours at x miles per hour, and b hours at y miles per hour; how long will it take him to finish the journey at z miles an hour?

11. Two places are x miles apart; two persons start at the same time, one from each place, and travel towards each other at the rate of a and b miles, respectively, per hour. How long will it be before they meet, and how far will each travel?

B.

1. A rectangular solid is a feet long, b feet wide and c feet thick. How many cubic feet are in it? How many square feet on all the faces? How many feet in the sum of the lengths of all the edges?

2. A room is x yards long and y feet wide; how many square inches are there in the area of the floor?

3. What will it cost to plaster the walls and ceiling of a room a feet long, b feet wide and c feet high at x cents per square yard?

4. A man works p hours a day for m days, and q hours a day for n days. He receives x cents per hour; how many dollars does he receive for this?

5. A grocer mixes a pounds of tea worth x cents a pound, with b pounds worth y cents a pound, and c pounds worth z cents a pound. What is a pound of the mixture worth?

6. In a class of n boys, x work at mathematics, y work at classics, and the rest are idle; what is the excess of workers over idlers?

If a and b denote two quantities, express algebraically :—

7.   That the square of the sum of two quantities is equal to the sum of their squares increased by twice their product.

8.   That the square of the difference between two quantities is equal to the sum of their squares diminished by twice their product.

9.   That the product of the sum and difference of two quantities is equal to the difference of their squares.

10.   That the cube of the sum of two quantities is equal to the sum of their cubes increased by three times their product into their sum.

11.   That the cube of the difference of two quantities is equal to the difference of their cubes diminished by three times their product into their difference.

12.   That the difference between the squares of the sum and difference of two quantities is equal to four times their product.

13.   That the sum of the squares of the sum and difference of two quantities is equal to twice the sum of their squares.

14.   That the sum of the cubes of two quantities divided by their sum is equal to the sum of their squares diminished by their product.

15.   That the difference between the cubes of two quantities divided by their difference is equal to the sum of their squares increased by their product.

16.   That the square of the difference between the squares of two quantities, increased by the square of twice their product, is equal to the square of the sum of their squares.

EXERCISE XVI.

PROBLEMS LEADING TO SIMPLE EQUATIONS.

A.

1.   Find two numbers whose sum is 38, and whose difference is 6.

2.   The sum of two numbers is 83 and their difference is 27 ; find the numbers.

3.   The difference of two numbers is 13 and their sum is 95 ; find them.

4.   One number exceeds another by 27, and their sum is 289 ; find them.

5.   Find a number such that, if 18 be added to it, twice the sum will be 114.

6.   Divide 60 into two such parts so that the double of one part may be three times as great as the other part.

7.   At an election where 975 votes were given the successful candidate had a majority of 63 ; how many voted for each ?

8.   The difference between two numbers is 26. When the larger is increased by 15 and the smaller by 12 their sum is 159 ; find the numbers.

9.   There are two numbers whose difference is 61. If the smaller is increased by 20 and the other decreased by 16 the sum is 295. What are the numbers ?

10.   To the double of a certain number I add 96 and obtain as a result 854. What is the number ?

11.   A father is 30 years old, and his son is 2 years old ; in how many years will the father be eight times as old as the son ?

12.   The sum of $216 was raised by A, B and C together ; B contributed $12 more than A, and C $27 more than B ; how much did C contribute ?

13.   Divide the number 63 into two parts such that three times one part may be equal to four times the other.

14.   Find three consecutive numbers whose sum shall be 255.

15.   Divide $2,426 among A, B and C so that A may have $23 more than B, and B $54 more than C.

16.   A is 36 years older than B, and A's age is as much above 15 as B's is below 55 ; find their ages.

17.   Twenty-nine times a certain number is as much above 99 as 189 is above 7 times the number ; find the number.

18. After 34 gal. had been drawn out of one of two equal casks, and 80 gal. out of the other, there remained just three times as much in one cask as in the other; What did each cask contain when full?

19. Divide $1,520 among A, B and C so that A may have $100 less than B, and B $270 less than C.

20. A number consists of two digits whose sum is 11. The tens digit is 3 more than the other. Find the number.

B.

1. A person buys 4 horses; for the second he gives $48 more than for the first, for the third $24 more than for the second, and for the fourth $8 more than for the third. The sum paid for all was $920; how much did the fourth cost?

2. Divide 60 into two parts such that the difference between the greater and 64 may be equal to twice the difference between the less and 38.

3. A gentleman left $1,100 to be divided among four servants, A, B, C, D; of whom B was to have twice as much as A, C as much as A and B together, and D as much as C and B together; how much had B and D?

4. A starts on a journey 20 minutes before B; A walks at the rate of 4 miles an hour, and B at the rate of 4½ miles an hour; at what distance along the road will B overtake A?

5. The length of a room exceeds its breadth by 3 feet; if the length had been increased by 3 feet and the breadth diminished by 2 feet, the area would not have been altered; find the dimensions.

6. The length of a room exceeds its breadth by 8 feet; if each had been increased by 2 feet the area would have been increased by 60 square feet; find the original dimensions.

7. From a certain number 3 is taken, and the remainder is divided by 4; the quotient is then increased by 9 and divided by 6 and the result is 3; find the number.

8.   A man buys a horse and harness for $108 and the horse cost five times as much as the harness.   Find the cost of the horse.

9.   Divide 96 into four such parts that the first increased by 3, the second diminished by 3, the third multiplied by 3, and the fourth divided by 3, shall all give the same result.

10.   A has $240 and B has $96, and each loses a certain sum.   Then A has five times as much as B. What is the sum lost by each?

11.   The sum of two digits is 9.   Four times one of the numbers they form is equal to seven times the other number.   Find the digits.

12.   A workman was employed for 40 days on condition that for every day he worked he should receive $1.37, and for every day he was idle he should forfeit 50 cents.   At the end of the time he received $37.97 ; find the number of days he worked.

13.   A certain number of men and twice as many women were employed on a work for a week.   Each man received $1.45 and each woman 83 cents a day ; their total wages being $317.22.   How many were employed altogether?

14.   Find three consecutive numbers such that, if they be divided by 10, 17 and 26 respectively, the sum of the quotients will be 10.

15.   A man sold a horse for £35 and one half as much as he gave for it, and gained thereby ten guineas ; what did he pay for the horse?

FACTORS, MULTIPLES AND FRACTIONS.

EXERCISE XVII.

Find the Highest Common Factor of:

1. $15x^3$, $18x^2$.

2. $21a^5$, $28 a^4$.

6. $45m^3n^4$, $60m^2n^3$.

7. $20ab^2c$, $36a^3bc^4$.

3. a^4b, a^2b^3.

4. x^3y^2z, xyz^3.

5. $14x^2y^3$, $24x^3y$.

8. $17p^2q^3r$, $51p^3qr^4$.

9. abc, $2ab^2c^3$.

10. $3x^2z^2$, x^3yz^2.

11. $7a^2b^4c^6$, $14a^6b^4c^2$.

12. $35a^2b^3x^3y^4$, $49a^3b^2x^4y^3$.

13. $63ax^2$, $49ay^2$, $56az^2$.

14. $34ab^2c$, $51a^2bc$, $68abc^2$.

15. $8a^2x$, $6abxy$, $10acx^2y^3$.

16. $a^3x^2y^2$, b^3xy^3, c^3x^3y.

17. $24a^3b^2c^3$, $48a^2b^3c^3$, $64a^3b^3c^2$.

18. $100xy^2z$, $75x^2yz$, $50xy$.

19. a^2pxy, b^2qxy, c^2rxy.

20. $15a^4b^5c^2$, $25a^5b^4c^7$, $60a^3b^7c^6$.

21. $30a^2c^3b$, $35a^3c^2b^2$, $42a^3cb^5$.

22. $22p^2m^2n^2$, $33p^4mn$, $44p^5m$.

23. $36x^4y^5z^6$, $48x^6y^5z^4$, $60x^3y^3z^7$.

24. $15xy$, $16yz$, $17xyz$.

EXERCISE XVIII.

Reduce to lowest terms:

1. $\dfrac{8a^2}{12a^3}$

2. $\dfrac{8x^3}{36x^2}$.

3. $\dfrac{10a^2b^3}{24a^3b^2}$.

4. $\dfrac{4axy}{3abc}$.

5. $\dfrac{18x^5y^3z^2}{45x^3y^2z^4}$.

6. $\dfrac{7a^5b^7c^9}{21a^9b^6c^3}$.

9. $\dfrac{12a^4bc^3}{14a^3b^2c^2}$

10. $\dfrac{105m^3n^2p}{42m^2n^3p^2}$.

11. $\dfrac{12a^4b^2x}{18a^2b^4y}$.

12. $\dfrac{16a^4b^3c}{20a^3b^4d}$.

13. $\dfrac{9bcyz}{12abxy^2}$.

14. $\dfrac{12a^2}{16ab}$.

17. $\dfrac{12mn^2p}{15m^2np^2}$

18. $\dfrac{abc^2}{a^3b^2c}$.

19. $\dfrac{3x^2yz^3}{5xy^2z^2}$.

20. $\dfrac{mn^4pq}{m^2n^5p^2}$.

21. $\dfrac{39a^2b^4c^5}{52a^3b^3c^6}$.

22. $\dfrac{38k^2m^4p^3}{57k^3m^3p^4}$.

7. $\dfrac{51ay^2z}{34a^2y^2z^2}.$ 15. $\dfrac{2x^2yz}{5xy^2z}$ 23. $\dfrac{46x^4y^5z^6}{69x^3y^4z^5}.$

8. $\dfrac{6x^3y^2z^3}{8x^5y^8z^4}.$ 16. $\dfrac{8a^2b}{10b^2c}.$ 24. $\dfrac{abxy}{x^2y^2cd}.$

EXERCISE XIX.

Simplify the following :

1. $\dfrac{3x}{4y} \times \dfrac{7y}{9x}.$

2. $\dfrac{3a}{4b} \times \dfrac{2b}{6a}.$

3. $\dfrac{4x^2}{9y^2} \times \dfrac{3y}{2x}.$

4. $\dfrac{8a^2b^3}{45x^2y} \times \dfrac{15xy^2}{4a^3b^2}.$

5. $\dfrac{9x^2y^2z}{15a^2b^2c} \times \dfrac{20a^3b^2c}{12xy^2z}.$

6. $\dfrac{3ab}{2cd} \times \dfrac{c^2d^3}{ab^2}.$

7. $\dfrac{12a^2bc}{8ab^3} \times \dfrac{36ab^2}{48b^2c^3}.$

8. $\dfrac{5a^3x}{3yz} \times \dfrac{xyz^3}{15a^2bc}.$

9. $\dfrac{7a^3b^2}{9ax^2y} \times \dfrac{18x^2c}{15ac^2}.$

10. $\dfrac{15x^2yz^3}{16m^2n} \div \dfrac{5x^2yz^2}{8mn^2}.$

11. $\dfrac{39m^2n^3}{42p^2k^3} \div \dfrac{13mn^4}{35k^3p^4}.$

12. $\dfrac{10xyz^3}{a^2bc} \div \dfrac{5yz}{3a^2x}.$

13. $\dfrac{a^2}{bc} \times \dfrac{c^2}{ab} \times \dfrac{b^2}{ac}.$

14. $\dfrac{b^2c}{y^2z} \times \dfrac{a^2b}{z^2x} \times \dfrac{c^2a}{x^2y}.$

15. $\dfrac{7a^5b^4}{5c^2d^3} \times \dfrac{20ac}{42bd} \times \dfrac{4c^3d^2}{3a^4b^3}.$

16. $\dfrac{25pm}{14k^2n} \times \dfrac{70k^3m^2}{75n^2q^2} \times \dfrac{25n^3q}{4p^2m}.$

17. $\dfrac{51m^2p}{21x^2y} \times \dfrac{7y^3z}{17p^2n} \div \dfrac{m^2x^2}{pyz}.$

18. $\dfrac{27b^2}{81c} \times \dfrac{15c^2}{40d^3} \div \dfrac{abc}{14d^3}.$

19. $\dfrac{4c^2}{5d^3} \div \dfrac{16a^2b^2c^2}{15d^4} \times \dfrac{b^2}{3c}.$

20. $\dfrac{8ax^2}{7by} \div \dfrac{16bdx}{14aby} \times \dfrac{49cy^2}{64dx^3}.$

21. $\dfrac{p^2q^2y}{10x^2} \times \dfrac{36p^3q^3}{81mn} \div \dfrac{15mpx^5}{27n^2x^3y^2}.$

22. $\dfrac{a^3}{b^3} \times \dfrac{xy}{ab} \div \dfrac{ap}{bx}.$

23. $\dfrac{6c^3x}{5ab^2} \div \dfrac{28a^2c^2}{15b^2xy^2} \times \dfrac{7a^3}{4x^3y^2}.$

24. $\dfrac{18x^2yz}{25a} \times \dfrac{15abc}{16xyz} \div \dfrac{100a^2bc}{128x^3y^2z^2}$

EXERCISE XX.

Find the Least Common Multiple of:

1. $4a^2b$, $6ab^2$.
2. $18a^3b^2c$, $12ab^2c^3$.
3. $8b^2x^2y^3$, $12a^2x^3y^2$.
4. $6a^3x$, $4a^2x^2$.
5. $3x^5y$, $12xy^3$.
6. $4a^2b^3c$, $8a^3b^3c^4$.
7. abc, $2a^2$.
8. $12xy$, $8ab$.
9. $5a^3b^4c^5$, $3a^4b^3c^2$.
10. $4x^3yz$, $6x^3y^3$.
11. $57a^3x^2y$, $76b^2x^3$.
12. $27m^2n^2p$, $81m^3nq$.
13. $8a^2$, $4ab$, $6bc^2$.
14. a^2bc, b^2ca, c^2ab.
15. $4x^3y^4$, $3x^2y^2$, $2xy^3$.
16. $7x^4y^2$, $2x^3y^3$, $3xy^5$.
17. $30a^2c^3b$, $35a^3c^2b^2$, $42b^3ca^2$.
18. $8a^2b^2$, $10b^2c^2$, $12abc$.
19. $44a^4b^2$, $33b^2c^3$, $22c^3ab^4$.
20. $35axy$, $40bxz$, $45cyz$.
21. $5mnp$, $4npq$, $3mpq$.
22. $39a^2b$, $52b^2c$, $65c^2a$.
23. $34x$, $51y$, $68z$.
24. $76ax^2$, $95xy^3$, $114bz^2$.

EXERCISE XXI.

Simplify the following:

1. $\dfrac{x}{3} + \dfrac{x}{4}$.

2. $\dfrac{a}{2} + \dfrac{a}{3}$.

3. $\dfrac{x}{5} + \dfrac{y}{7}$.

4. $\dfrac{2x}{5} + \dfrac{4x}{7}$.

5. $\dfrac{a}{13} + \dfrac{b}{39}$.

6. $\dfrac{2a}{9} - \dfrac{3a}{14}$.

7. $\dfrac{6x}{5} - \dfrac{3x}{4}$.

11. $\dfrac{x}{2} - \dfrac{x}{3} + \dfrac{x}{5}$.

12. $\dfrac{a}{4} - \dfrac{a}{6} + \dfrac{a}{12}$.

13. $\dfrac{x}{3} - \dfrac{x}{9} + \dfrac{x}{6}$.

14. $\dfrac{2x}{3} - \dfrac{3x}{4} + \dfrac{5x}{6}$.

15. $\dfrac{8x}{9} - \dfrac{5x}{12} + \dfrac{7x}{6}$.

16. $\dfrac{15x}{16} - \dfrac{3x}{8} - \dfrac{x}{12}$.

17. $\dfrac{23x}{27} - \dfrac{5x}{18} - \dfrac{2x}{9}$.

21. $a + \dfrac{y}{b}$.

22. $\dfrac{3x}{2a} + \dfrac{4y}{3b}$.

23. $\dfrac{7a}{12x} - \dfrac{5b}{24y}$.

24. $a - \dfrac{by}{cx}$.

25. $2x^2 - \dfrac{b^2}{a}$.

26. $\dfrac{2x}{a^2c^2} - \dfrac{3y}{a^3c^4}$.

27. $\dfrac{3a}{7b} + \dfrac{5b}{21c}$.

8. $\dfrac{5m}{12} - \dfrac{5m}{36}$.

9. $\dfrac{7y}{8} - \dfrac{5y}{6}$.

10. $\dfrac{11ab}{12} - \dfrac{5ab}{8}$.

18. $\dfrac{5a}{8} + \dfrac{3b}{9} + \dfrac{7c}{12}$.

19. $\dfrac{a+b}{14} + \dfrac{a-b}{21}$.

20. $\dfrac{x-y}{18} + \dfrac{x+y}{27}$.

28. $\dfrac{2x}{3y} - 4c$.

29. $\dfrac{m}{3x} + \dfrac{n}{2x} - \dfrac{p}{x}$.

30. $\dfrac{a}{4n} - \dfrac{b}{5n} - \dfrac{c}{6n}$.

ON WRITING PRODUCTS AND FACTORS.

EXERCISE XXII.

Write the product of :

1. $x+7$ and $x+10$.
2. $x+8$ and $x+9$.
3. $x+5$ and $x+12$.
4. $a+9$ and $a+8$.
5. $a+11$ and $a+3$.
6. $a+2$ and $a+17$.
7. $m+1$ and $m+2$.
8. $m+12$ and $m+4$.
9. $m+11$ and $m+10$.
10. $c+13$ and $c+1$.

11. $(x+2y)(x+3y)$.
12. $(a+4b)(a+7b)$.
13. $(a+9x)(a+12x)$.
14. $(b+9c)(b+10c)$.
15. $(c+16y)(c+y)$.
16. $(x+z)(x+10z)$.
17. $(2x+7)(2x+8)$.
18. $(2a+5)(2a+4)$.
19. $(2x+11)(2x+7)$.
20. $(2a+3)(2a+2)$.

Write the expressions which multiplied give :

21. $x^2 + 10x + 16$.
22. $x^2 + 12x + 27$.
23. $x^2 + 13x + 42$.
24. $x^2 + 13x + 40$.
25. $a^2 + 12a + 27$.
26. $a^2 + 11a + 28$.
27. $a^2 + 17a + 60$.
28. $y^2 + 14y + 13$.
29. $y^2 + 16y + 28$.
30. $y^2 + 15y + 36$.

31. $x^2 + 8xy + 7y^2$.
32. $x^2 + 12xy + 35y^2$.
33. $x^2 + 10xy + 21y^2$.
34. $a^2 + 10ab + 24b^2$.
35. $a^2 + 10ab + 9b^2$.
36. $a^2 + 15ab + 44b^2$.
37. $x^2 + 14xz + 48z^2$.
38. $a^2 + 7ac + 10c^2$.
39. $a^2 + 7ax + 12x^2$.
40. $x^2 + 17xy + 72y^2$.

EXERCISE XXIII.

Write the product of:

1. $(x-4)(x-8)$.
2. $(x-4)(x-9)$.
3. $(a-10)(a-4)$.
4. $(a-8)(a-7)$.
5. $(c-5)(c-9)$.
6. $(c-2)(c-5)$.
7. $(2x-7)(2x-5)$.
8. $(2x-9)(2x-3)$.
9. $(3a-5)(3a-4)$.
10. $(3a-8)(3a-2)$.

11. $(x-7y)(x-6y)$.
12. $(x-6y)(x-8y)$.
13. $(x-9y)(x-6y)$.
14. $(a-2b)(a-8b)$.
15. $(a-10b)(a-2b)$.
16. $(a-2c)(a-15c)$.
17. $(2a-3b)(2a-5b)$.
18. $(2x-4y)(2x-8y)$.
19. $(3a-5c)(3a-8c)$.
20. $(3m-6n)(3m-9n)$.

Write the expressions which multiplied give :

21. $x^2-15x+56$.
22. $x^2-17x+72$.
23. $x^2-18x+80$.
24. $a^2-11a+18$.
25. $a^2-12a+27$.
26. $a^2-16a+63$.

27. $m^2-6mn+5n^2$.
28. $p^2-9pq+8q^2$.
29. $c^2-4cx+3x^2$.
30. $x^2-10xy+21y^2$.
31. $x^2-12xy+27y^2$.
32. $a^2-15ab+36b^2$.

EXERCISE XXIV.

Write the product of:

1. $(x+9)(x-7)$.
2. $(x+8)(x-5)$.
3. $(x+7)(x-2)$.
4. $(a+6)(a-3)$.
5. $(a+9)(a-4)$.
6. $(a+8)(a-3)$.
7. $(m+6)(m-2)$.
8. $(n+10)(n-3)$.
9. $(p+9)(p-6)$.
10. $(b+12)(b-7)$.

11. $(x-12)(x+3)$.
12. $(x-11)(x+6)$.
13. $(x-10)(x+8)$.
14. $(a-12)(a+5)$.
15. $(a-11)(a+7)$.
16. $(b-12)(b+9)$.
17. $(b-10)(b+2)$.
18. $(c-9z)(c+3z)$.
19. $(c-10d)(c+d)$.
20. $(x-17y)(x+5y)$.

Write the expressions which multiplied give :

21. $x^2 + 9x - 36$.
22. $x^2 + 7x - 44$.
23. $x^2 + 5xy - 50y^2$.
24. $a^2 + 7a - 18$.
25. $a^2 + 5a - 24$.
26. $a^2 + 3ac - 28c^2$.
27. $c^2 + 5c - 6$.
28. $c^2 + 11cd - 26d^2$.
29. $m^2 + 11mn - 42n^2$.
30. $n^2 + 8ny - 105y^2$.

31. $x^2 - 5x - 84$.
32. $x^2 - 3x - 88$.
33. $x^2 - 4xy - 60y^2$.
34. $a^2 - 10a - 39$.
35. $a^2 - 18ab - 40b^2$.
36. $y^2 - 24y - 25$.
37. $y^2 - 21yz - 72z^2$.
38. $c^2 - 13cy - 48y^2$.
39. $z^2 - 38z - 80$.
40. $b^2 - 16bc - 17c^2$.

EXERCISE XXV.

Write the square of :

1. $x + y$.
2. $a + b$.
3. $m + n$.
4. $a + x$.
5. $a + c$.
6. $x + z$.
7. $b + 8$.
8. $c + 10$.
9. $d + 12$.
10. $2x + 3y$.

11. $a + 3b$.
12. $x + 5y$.
13. $5ab + c$.
14. $p - q$.
15. $x - ab$.
16. $xy - z$.
17. $ab - c$.
18. $ax - 2b$.
19. $pq - r$.
20. $7a - 4b$.

21. $3xy - 9$.
22. $9mn - 3c$.
23. $4a + 6ab$.
24. $12x - 11y$.
25. $7x + 10z$.
26. $13c - 1$.
27. 1009.
28. 1012.
29. 688.
30. 989.

Write what each is the square of :

31. $c^2 + 2cd + d^2$.
32. $x^2 + 4xy + 4y^2$.
33. $81a^2 + 18ab + b^2$.
34. $9x^2 + 12xy + 4y^2$.
35. $1 + 2a + a^2$.
36. $25p^2q^2 + 60pqr + 36r^2$.

37. $x^2 - 10xy + 25y^2$.
38. $4a^2 - 12ab + 9b^2$.
39. $25p^2 - 30pq + 9q^2$.
40. $36a^2 - 84ac + 49c^2$.
41. $49b^2 - 14b + 1$.
42. $16x^2y^2 - 96xy + 144$.

EXERCISE XXVI.

Add a term to each of the following and state what each is the square of:

1. $b^2 + 2bc$.
2. $c^2 + 6c$.
3. $p^2 + 2pq$.
4. $p^2 + 10pq$.
5. $x^2 + 2x$.
6. $a^2b^2 + 6ab$.
7. $y^2 - 8y$.
8. $x^2 - 12x$.
9. $a^2 - 14ab$.
10. $x^2 - 20xz$.

11. $4a^2 + 12ab$.
12. $9p^2 + 24pq$.
13. $16x^2 + 24x$.
14. $y^2 + 38y$.
15. $36a^2 + 12a$.
16. $25a^2 + 20ab$.
17. $4a^2 + 4ac$.
18. $81x^2 - 18xy$.
19. $9a^2 - 12ab$.
20. $4c^2 - 12cd$.

21. $25x^2 - 30xy$.
22. $1 - 2a$.
23. $a^2 - 4ax$.
24. $4a^2 x^2 - 28acx$.
25. $9a^2m^2 + 30amxy$.
26. $16c^2 - 8c$.
27. $4x^2y^2 + 12xy$.
28. $a^2x^2 - 2ax$.
29. $b^2y^2 + 4by$.
30. $169x^2z^2 - 52xz$.

EXERCISE XXVII.

Write the product of:

1. $(x+y)(x-y)$.
2. $(a+b)(a-b)$.
3. $(a+x)(a-x)$.
4. $(x+z)(x-z)$.
5. $(b+c)(b-c)$.
6. $(x+8)(x-8)$.
7. $(a+12)(a-12)$.
8. $(2x+3y)(2x-3y)$.
9. $(2m+4n)(2m-4n)$.
10. $(3a+5b)(3a-5b)$.

11. $(1+x)(1-x)$.
12. $(1+9a)(1-9a)$.
13. $(2+7c)(2-7c)$.
14. $(ab+3c)(ab-3c)$.
15. $(12p+10q)(12p-10q)$.
16. $(7ab+9x)(7ab-9x)$.
17. $(12x+6by)(12x-6by)$.
18. $(9xy+1)(9xy-1)$.
19. $(3ab+7cd)(3ab-7cd)$.
20. $(a^2+b^2)(a^2-b^2)$.

21. $(x+y)+z$ and $(x+y)-z$.
22. $a+b+c$ and $a+b-c$.
23. $l+m+n$ and $l+m-n$.
24. $2a+3b+4c$ and $2a+3b-4c$.
25. $5x+2y+3z$ and $5x-2y+3z$.
26. $3a+4b+c$ and $3a-4b+c$.
27. x^2+x+1 and x^2-x+1.
28. a^2x^2+ax+1 and a^2x^2-ax+1.
29. x^2+xy+y^2 and x^2-xy+y^2.
30. a^2+ab+b^2 and a^2-ab+b^2.

EXERCISE XXVIII.

Write the factors of :

A.

1. $x^2 - 9$.
2. $x^2 - 100$.
3. $y^2 - 81$.
4. $a^2 - 144$.
5. $9 - a^2 b^2$.
6. $64 - c^2$.
7. $121 - y^2$.
8. $225 - x^2 y^2$.
9. $x^2 - 16a^2$.
10. $b^2 - 25c^2$.
11. $36x^2 - 25z^2$.
12. $1 - 9a^2$.

13. $49p^2 - 36q^2$.
14. $4m^2 - 1$.
15. $81 - 100k^2$.
16. $25x^2 - 4$.
17. $a^2 - 9b^2$.
18. $16x^2 - y^2$.
19. $a^2 b^2 - 49$.
20. $x^2 - 81y^2 z^2$.
21. $a^2 b^2 - 4c^2 d^2$.
22. $25a^2 - 64$.
23. $p^2 q^2 - 144$.
24. $x^4 - 9y^2$.

25. $x^2 - 16b^4$.
26. $100a^2 - 1$.
27. $121x^2 - 81z^2$.
28. $p^2 q^2 - 49a^4$.
29. $64x^4 - 25y^6$.
30. $121x^4 - y^4$.
31. $16x^6 - 9y^{16}$.
32. $a^2 b^2 c^2 - x^2 y^2$.
33. $25x^{10} - 16c^4$.
34. $1 - 100x^4 y^6 z^8$.
35. $m^2 - 4$.
36. $m^2 n^8 - 225p^6$.

B.

1. $(a-b)^2 - c^2$.
2. $(x+y)^2 - z^2$.
3. $(a-b)^2 - c^2$.
4. $(x-y)^2 - z^2$.
5. $(a+2b)^2 - 9c^2$.
6. $(x-2y)^2 - 16z^2$.
7. $(x+5a)^2 - 25b^2$.
8. $(3x-4a)^2 - 36c^2$.
9. $(a-3x)^2 - y^2$.
10. $(9x+3c)^2 - 4$.
11. $9a^2 - (3b+4c)^2$.
12. $c^2 - (5a-2b)^2$.
13. $x^2 - (y+z)^2$.
14. $1 - (3x-4z)^2$.
15. $(a+b)^2 - (c+d)^2$.

16. $(a+n)^2 - (b-m)^2$.
17. $(3a+2x)^2 - (2b+3y)^2$.
18. $(a-2b)^2 - (3c-4d)^2$.
19. $(4x+a)^2 - (b+4y)^2$.
20. $1 - (7a-5x)^2$.
21. $(a+b+c)^2 - 1$.
22. $(x-y+z)^2 - 289$.
23. $(x-5y)^2 - 81z^2$.
24. $(x+y)^2 - (x-y)^2$.
25. $(x+3y)^2 - 4y^2$.
26. $(21x+3y)^2 - (18x-4y)^2$.
27. $(9x+3)^2 - (6x-8)^2$.
28. $(7a+1)^2 - (2a-1)^2$.
29. $25a^2 - (4a+1)^2$.
30. $(2x-3y+4z)^2 - (6y-5z)^2$.

C.

1. $(a^2 + 2ab + b^2) - c^2$.
2. $(x^2 - 2xy + y^2) - z^2$.
3. $a^2 - 2ab + b^2 - x^2$.
4. $a^2 - (b^2 - 2bc + c^2)$.

8. $x^2 + 2ax + a^2 - y^2$.
9. $y^2 - c^2 + 2cx - x^2$.
10. $x^2 - 4xy + 4y^2 - 9x^2 y^2$.
11. $a^2 - 10ab + 25b^2 - 1$.

5. $1 - m^2 - 2mn - n^2$.   12. $16x^2 - a^2 - 6ab - 9b^2$.
6. $x^2 + 2xy + y^2 - 9x^2y^2$.   13. $4m^2 - p^2 + 2pq - q^2$.
7. $a^2 - 2ab + b^2 - 4a^2b^2$.   14. $p^2 + 4pq + 4q^2 - r^2$.
15. $x^2 - y^2 + 2xz + z^2$.
16. $(a^2 + 2ab + b^2) - (c^2 - 2cd + d^2)$.
17. $a^2 - 2ab + b^2 - c^2 - 2cd - d^2$.
18. $(2a + 3b)^2 - (3b + 4c)^2$.
19. $x^2 - 2xy + y^2 - b^2 + 4bc - 4c^2$.
20. $a^2 + 6ax + 9x^2 - y^2 - 2by - b^2$.
21. $x^2 - 2x + 1 - m^2 - 4mn - 4n^2$.
22. $a^2 + b^2 + 2ab - c^2 - d^2 - 2cd$.
23. $a^2 - b^2 + c^2 - d^2 - 2ac + 2bd$.
24. $a^2 + x^2 - y^2 - z^2 + 2ax - 2yz$.
25. $a^2 - 8ab + 16b^2 - 1 + 10c - 25c^2$.
26. $x^2 + 6abx + 9a^2b^2 - y^2 + 8cdy - 16c^2d^2$.
27. $x + y^2 - z^2 - w^2 - 2xy - 2zw$.
28. $4a^2b^2 + 12abcd + 9c^2d^2 - 16x^2y^2$.
29. $25a^2b^2 - 36c^2 + 10acd - 81d^2$.
30. $4x^2 - c^2 - d^2 + 9a^2 - 12ax - 2cd$.

EXERCISE XXIX.

Find the value of:

1. $(675)^2 - (325)^2$.
2. $(731)^2 - (631)^2$.
3. $(650)^2 - (350)^2$.
4. $(1613)^2 - (387)^2$.
5. $(1922)^2 - (578)^2$.
6. $(4632)^2 - (368)^2$.
7. $(5187)^2 - (987)^2$.
8. $(1001)^2 - (999)^2$.
9. $(8176)^2 - (8171)^2$.
10. $(9876)^2 - (9864)^2$.

EXERCISE XXX.

Write the square of:

1. $a + b + c$.
2. $x + y + z$.
3. $m + n + p$.
4. $a + b - c$.
5. $x - y + z$.
6. $a - b - c$.
7. $x + 2y + z$.
8. $3a - 4b + 2c$.
9. $1 + 2x - 3y$.
10. $5 - 8y + 6z$.
11. $4 - 5x - 6x^2$.
12. $3x^2 + 7x - 8$.
13. $5p - 4q + 9r$.
14. $x^2 + y^2 + z^2$.
15. $a^2 - b^2 - c^2$.
16. $xy + yz + zx$.
17. $5x^2 - 9x + 3$.
18. $1 - ax - by$.

19. Find the value of $(x+y+z)^2 + (x+y-z)^2 + (y+z-x)^2 + (z+x-y)^2$.
20. Find the value of $(a+b+c)^2 + (a+b-c)^2 + (b+c-a)^2 + (c+a-b)^2$.
21. Without multiplication find the continued product of $(x+y+z)(x+y-z)(y+z-x)(z+x-y)$.
22. Find the product of $(a+b+c)(a+b-c)(b+c-a)(c+a-b)$.
23. Find the value of $x^2+y^2+z^2-xy-yz-zx$, when $x=a+b$, $y=b+c$, $z=c+a$.
24. Find the value of $x^2+y^2+z^2+xy+yz+zx$, when $x=a+b+c$, $y=a-b+c$, $z=a+b-c$.
25. Of what expression is $25x^2 - 60xy + 36y^2 - 70xz + 84yz + 49z^2$ the square?

EXERCISE XXXI.

Write the cube of:

A.

1. $x+y$.	6. $x-3$.	11. $3x+y$.	16. $5x^2-4y^2$.
2. $a+b$.	7. $a+4$.	12. $4x-5y$.	17. $3a^2-2b^2$.
3. $x+a$.	8. $a-5$.	13. $ab+c$.	18. $5x^3+1$.
4. $x-a$.	9. $x+2y$.	14. $2xy-3z$.	19. $7a^2-1$.
5. $x+2$.	10. $a-2b$.	15. $6a+bc$.	20. $ax-y^2$.

B.

1. $a+(b+c)$.	7. $2x+3y+4z$.	13. $x^2+y^2+z^2$.
2. $(x+y)+z$.	8. $4a-3b+2c$.	14. $a^3-b^3-c^3$.
3. $(a+b)-c$.	9. $5x-4y-7z$.	15. $2x^2-5y^2+4z^2$.
4. $x+y-z$.	10. $6a-5b-4c$.	16. $ax-by+cz$.
5. $x-y+z$.	11. $3p+4q-8r$.	17. $a+b+c+d$.
6. $a-b+c$.	12. $3a-2b+5x$.	18. $2x+3y+4z+5w$.

EXERCISE XXXII.

Write the product of:

1. $(x+a)(x+b)(x+c)$.
2. $(x+a)(x-b)(x+c)$.
3. $(x+a)(x-b)(x-c)$.
9. $(2x+3)(2x+5)(2x+7)$.
10. $(2x-4)(2x-6)(2x-8)$.
11. $(3x+1)(3x+5)(3x+7)$.

4. $(x+1)(x+2)(x+3)$.   12. $(3x-8)(3x+6)(3x-5)$.
5. $(x+4)(x+5)(x+6)$.   13. $(4a+2)(4a+3)(4a-9)$.
6. $(a-3)(a-7)(a-2)$.   14. $(x+4)(x+5)(x+2)(x+3)$.
7. $(x+6)(x+1)(x+7)$.   15. $(a+5)(a+1)(a+6)(a+8)$.
8. $(x-9)(x+8)(x-5)$.   16. $(x-9)(x+8)(x-7)(x+6)$.

ON WRITING QUOTIENTS AND FACTORS.

EXERCISE XXXIII.

Write the quotient of:

A.

1. $x^3+y^3 \div x+y$.
2. $a^3+b^3 \div a+b$.
3. $x^3+a^3 \div x+a$.
4. $b^3+y^3 \div b+y$.
5. $x^3+1 \div x+1$.
6. $a^3+1 \div a+1$.
7. $1+c^3 \div 1+c$.
8. $1+z^3 \div 1+z$.
9. $x^3+8 \div x+2$.
10. $a^3+27 \div a+3$.
11. $x^3+8y^3 \div x+2y$.
12. $27a^3+b^3 \div 3a+b$.
13. $8x^3+27b^3 \div 2x+3b$.
14. $x^3-y^3 \div x-y$.
15. $a^3-b^3 \div a-b$.

16. $a^3-125 \div a-5$.
17. $216-x^3 \div 6-x$.
18. $64x^3-125y^3 \div 4x-5y$.
19. $m^5+n^5 \div m+n$.
20. $a^5+b^5 \div a+b$.
21. $x^7+y^7 \div x+y$.
22. $a^7-b^7 \div a-b$.
23. $m^7-n^7 \div m-n$.
24. $x^5+1 \div x+1$.
25. $c^5-1 \div c-1$.
26. $a^5+32 \div a+2$.
27. $x^5-243 \div x-3$.
28. $x^6+a^9 \div x^2+a$.
29. $(a+b)^3+c^3 \div a+b+c$.
30. $(x-y)^3-z^3 \div x-y-z$.

B.

1. $x^4-y^4 \div x-y$.
2. $a^6-b^6 \div a-b$.
3. $m^8-n^8 \div m-n$.
4. $x^6-1 \div x-1$.
5. $1-a^8 \div 1-a$.

6. $16x^4-81y^4 \div 2x+3y$.
7. $625a^4-256b^4 \div 5a+4b$.
8. $x^6-y^6 \div x+y$.
9. $a^4-16b^4 \div a+2b$.
10. $(a+b)^4-1 \div a+b+1$.

EXERCISE XXXIV.

Write the expressions which multiplied give :

1. $x^3 + y^3$.
2. $x^3 - y^3$.
3. $a^3 - b^3$.
4. $a^3 + 1$.
5. $1 + a^3$.
6. $8x^3 - y^3$.
7. $a^3 + 8b^3$.
8. $27x^3 + 8$.

9. $a^4 - c^4$.
10. $a^8 - 1$.
11. $p^6 - q^6$.
12. $x^6 - 729$.
13. $a^3b^3 + c^3$.
14. $27x^3 - 8y^3$.
15. $c^3 - 216$.
16. $125 + z^3$.

17. $x^3y^3 + 512$.
18. $343a^3 - 1$.
19. $125x^3 - 1000y^3$.
20. $x^7 - y^7$.
21. $a^5 + b^5$.
22. $n^7 + 1$.
23. $1 - c^4$.
24. $p^3q^3 + 27r^3$.

RESOLUTION INTO FACTORS.

EXERCISE XXXV.

Resolve into factors :

A.

1. $5x^2 - 15x$.
2. $a^3 - a^2$.
3. $3x^2 - 6x^3$.
4. $8x - 4x^2$.
5. $7a^2x - 14ax^2$.

6. $6p^2 + p$.
7. $x^2 + xy$.
8. $x^5 - x^4y^2$.
9. $3x^2y^2 - 9xy^4$.
10. $16 + 24x^2$.

11. $12ab - 8$.
12. $34 + 51x^2y$.
13. $15a^4 - 225a^2$.
14. $25x^3y - 10x^3$.
15. $24a^2 - 27ab^2$.

16. $12a^3b - 9ab^2$.
17. $15x^4y - 6x^2y^3$.
18. $8m^3n^2p + 20m^2n^3p$.
19. $x^3 + x^2y - xy^2$.
20. $4x^2y^3 - 6x^2y^2 + 4x^3y^2$.
21. $15x^5 - 10a^2x^3 - 5a^3x^4$.
22. $38a^2x^5 + 76a^3x^4 + 57a^4x^3$.
23. $3a^3b - 3a^2b^2 + 3ab^3$.

24. $20ax^2 - 40ax + 45a$.
25. $6x^3 - 9x^4 + 4x^6$.
26. $35m^2x^2 - 70mx^2 + 105x^3$.
27. $7a - 14a^2 + 7a^4$.
28. $22m^2 - 33mn + 44n^2$.
29. $39p^2q^2 + 26pq - 52p^3$.
30. $ax^3 - abx^2 + acx$.
31. $14a^2b^3c^4 + 7a^3b^4c^2 - 21a^4b^2c^3$.

B.

1. $a^2 + ab + ac + bc$.
2. $a^2 - ac + ab - bc$.
3. $x^2 + xy + xz + yz$.

11. $x^3 + x^2 + x + 1$.
12. $mx + ma - nx - na$.
13. $15ax - 10bx - 12ay + 8by$.

4. $x^2 - xy - xz + yz$.
5. $a^2b^2 + abc + abd + cd$.
6. $a^2c^2 - abc - acd + bd$.
7. $a^2 + 4a + ab + 4b$.
8. $mx + my - nx - ny$.
9. $2ax - 2bx + 3ay - 3by$.
10. $pr - qr + ps - qs$.

14. $2x^2 - 10x - xy + 5y$.
15. $3ax^2 + 2axy + 3bxy + 2by^2$.
16. $axy - bcxy - az + bcz$.
17. $a^2x^2 + a^2y^2 + b^2x^2 + b^2y^2$.
18. $y^3 - y^2 + y - 1$.
19. $x^4 + x^3 + 3x + 3$.
20. $ax + bx - by - cy + cx - ay$.

EXERCISE XXXVI.

Resolve into factors:

1. $x^2 + 3x + 2$.
2. $a^2 + 7a + 12$.
3. $x^2 + 20x + 96$.
4. $a^2 + 23a + 102$.
5. $p^2 + 30p + 216$.
6. $x^2y^2 + 31xy + 130$.
7. $77 + 18x + x^2$.
8. $x^2 + 13x + 42$.
9. $x^2 + 5xy + 6y^2$.
10. $a^2 + 9ab + 14b^2$.
11. $x^2 + 49xy + 600y^2$.
12. $m^2 + 43mn + 390n^2$.
13. $p^2 + 35pq + 216q^2$.
14. $a + 5a^2 + 6a^3$.
15. $3x^2 + 45x + 168$.

16. $x^2 - 11x + 30$.
17. $x^2 - 19x + 90$.
18. $x^2 - 15x + 56$.
19. $a^2 - 21a + 110$.
20. $a^2 - 18a + 45$.
21. $a^2 - 21a + 104$.
22. $m^2 - 22mn + 105n^2$.
23. $x^2y^2 - 17xy + 52$.
24. $x^2y^2 - 23xy + 132$.
25. $a^2 - 20ab + 91b^2$.
26. $a^2 - 21abc + 90b^2c^2$.
27. $110 - 21x + x^2$.
28. $a^2x^2 - 24ax + 143$.
29. $7x^2 - 105x + 392$.
30. $a^2x^2 - 21ax + 98$.

B.

1. $x^2 + 11x - 80$.
2. $x^2 + x - 110$.
3. $a^2 + 16a - 260$.
4. $a^2x^2 + 14ax - 240$.
5. $x^2 + 7xy - 60y^2$.
6. $a^2 + ab - 42b^2$.
7. $m^2 + m - 156$.
8. $a^2 + 12abx - 45b^2x^2$.
9. $x^4 + 8a^2x^2 - 216a^4$.
10. $3ax^2 + 36axy - 255ay^2$.

11. $x^2 - 9x - 90$.
12. $x^2 - 11x - 152$.
13. $a^2 - 32ab - 105b^2$.
14. $a^2 - 20ab - 96b^2$.
15. $m^2 - 11m - 26$.
16. $m^2 - mn - 56n^2$.
17. $x^2y^2 - 3xyz - 10z^2$.
18. $165 - 4ax - a^2x^2$.
19. $420 - a - a^2$.
20. $5a^2x - 50ax^2y - 195x^3y^2$.

EXERCISE XXXVII.

Express iu the form of a square:

1. $x^2 + 18x + 81$.
2. $a^2 + 26a + 169$.
3. $m^2 + 34m + 289$.
4. $y^2 + 2y + 1$.
5. $z^2 + 20z + 100$.
6. $x^4 + 14x^2 + 49$.
7. $x^2 + 12xy + 36y^2$.
8. $m^2 + 22mn + 121n^2$.
9. $x^6 + 24x^3 + 144$.
10. $a^2 - 36a + 324$.
11. $p^2 - 32pq + 256q^2$.
12. $a^2 - 30ab + 225b^2$.
13. $4a^2 - 12ab + 9b^2$.
14. $a^2 - 4ax + 4x^2$.
15. $4a^2x^2 - 28acx + 49c^2$.
16. $9a^2m^2 + 30amxy + 25x^2y^2$.
17. $16a^2x^4 - 8ab^2c^3x^2 + b^4c^6$.
18. $16x^2y^2 + 24xyz + 9z^2$.
19. $1 - 2x^3 + x^6$.
20. $9a^2 - 12a + 4$.
21. $a^2 - 2abc + c^2b^2$.
22. $25 - 40n + 16n^2$.
23. $a^4x^2 + 4a^2x^2y^2 + 4x^2y^4$.
24. $49x^6 + 42x^3yz^2 + 9y^2z^4$.
25. $a^4b^4 - a^2b^2 + \frac{1}{4}$.
26. $3a^2b^2 - 18ab + 27$.
27. $(a+b)^2 + 2(a+b)c + c^2$.
28. $(x+y) - 2(x+y)z + z^2$.
29. $(a-b)^2 + 10(a-b) + 25$.
30. $(2m - 3n)^2 - 8(2m - 3n)p + 16p^2$.

EXERCISE XXXVIII.

Resolve into factors:

1. $x^4 + x^2 + 1$.
2. $a^4 + a^2 + 1$.
3. $x^4 + 10x^2 + 49$.
4. $x^4 + 2x^2 + 81$.
5. $a^4 + 25a^2 + 625$.
6. $m^4 + 4m^2 + 16$.
19. $4a^4 + 7a^2b^2 + 16b^4$.
20. $x^4 + 4y^4$.
21. $a^4 + 4b^4c^4$.
22. $x^8 + x^4y^4 + y^8$.
23. $x^8 + x^4 + 1$.
24. $4a^4 - 101a^2b^2 + 25b^4$.

7. $a^4 - 18a^2b^2 + b^4$.
8. $x^4 + x^2y^2 + y^4$.
9. $4a^4 - 8a^2 + 1$.
10. $81c^4 + 9c^2d^2 + d^4$.
11. $x^4 + 2x^2y^2 + 9y^4$.
12. $9x^4 + 2x^2y^2 + 1$.
13. $a^4 + a^2y^2 + y^4$.
14. $x^4 + 3x^2y^2 + 4y^4$.
15. $a^4 + 2a^2x^2 + 9x^4$.
16. $a^4 + 4a^2x^2 + 16x^4$.
17. $x^4 + x^2 + 25$.
18. $a^4 + 3a^2 + 36$.

25. $36x^4 - 97x^2y^2 + 36y^4$.
26. $144m^4 - 289m^2n^2 + 100n^4$.
27. $4c^4 + 1$.
28. $9p^4 - 10p^2q^2 + q^4$.
29. $x^4 - 113x^2 + 16$.
30. $9x^4 - 73x^2y^2 + 16y^4$.
31. $a^4 + 5a^2c^2 + 49c^4$.
32. $x^4 + 7x^2y^2 + 64y^4$.
33. $m^4 - 7m^2 + 81$.
34. $x^4 - 7x^2 + 1$.
35. $a^4 - 14a^2 + 1$.
36. $x^4 - 5x^2y^2 + 4y^4$.

EXERCISE XXXIX.

Resolve into factors:

1. $x^6 - y^6$.
2. $a^6 - b^6$.
3. $a^6b^6 - 1$.
4. $x^3y^3 + z^3$.
5. $8x^3 + a^6$.
6. $x^6 - 8a^3$.
7. $x^6 + y^6$.
8. $a^6 + b^6$.
9. $27a^3 - 64b^3$.
10. $p^3q^3 - 27r^3$.
11. $x^5 + 32y^5$.
12. $x^9 - y^9$.
13. $m^{10} - b^{10}$.
14. $x^{32} - y^{32}$.
15. $a^{12} - b^{12}$.

16. $8a^3b^3 + 125c^3$.
17. $x^3y^3z^3 - 1$.
18. $(a+b)^3 + c^3$.
19. $(x-y)^3 - z^3$.
20. $(a-b)^3 - 1$.
21. $1 - (x+y)^3$.
22. $a^4 - (b+c)^4$.
23. $a^3 + b^3 + a + b$.
24. $x^3 + y^3 + 3xy(x+y)$.
25. $x^3 - y^3 - 3xy(x-y)$.
26. $(c+d)^3 + (c-d)^3$.
27. $729a^7b - ab^7$.
28. $a^8x^6 - 64a^2y^6$.
29. $a^3 + 3a^2b + 3ab^2 + b^3 - c^3$.
30. $x^3 - 3x^2y + 3xy^2 - y^3 - z^3$.

EXERCISE XL.

Resolve into factors:

A.

1. $3x^2 + 5x + 2$.
2. $2a^2 + 5a + 2$.
3. $3x^2 + 10x + 3$.

13. $6x^2 - 31x + 35$.
14. $6x^2 - 23x + 20$.
15. $6x^2 - 7x + 2$.

4. $4a^2 + 9a + 2.$
5. $3x^2 + 8x + 5.$
6. $5x^2 + 14x + 8.$
7. $2x^2 + 9x + 10.$
8. $3a^2 + 10a + 8.$
9. $7x^2 + 16x + 4.$
10. $4a^2 + 23a + 15.$
11. $3x^2 + 41x + 26.$
12. $8m^2 + 38m + 35.$

16. $8x^2 - 34xy + 21y^2.$
17. $4a^2 - 13a + 9.$
18. $7 - 10m + 3m^2.$
19. $4x^2 - 14x + 12.$
20. $3x^2 - 23x + 14.$
21. $56x^2 - 67xz + 20z^2.$
22. $24b^2 - 17bc + 3c^2.$
23. $24x^2 - 50xy + 21y^2.$
24. $56m^2 - 229mn + 20n^2.$

B.

1. $4x^2 + 11x - 3.$
2. $3a^2 + 13a - 30.$
3. $3x^2 + 14x - 5.$
4. $2m^2 + 15m - 8.$
5. $3c^2 + 7c - 6.$
6. $4p^2 + p - 14.$
7. $12a^2 + 17a - 7.$
8. $5 + 32y - 21y^2.$
9. $12x^2 + 13xy - 35y^2.$
10. $39a^2 + 13ab - 26b^2.$

11. $6x^2 - 7x - 3.$
12. $2a^2 - a - 15.$
13. $3a^2 - 19a - 14.$
14. $12b^2 - 31b - 15.$
15. $12x^2 - x - 35.$
16. $2x^2 - 5xy - 3y^2.$
17. $24a^2 - 29ab - 4b^2.$
18. $20 - 9c - 20c^2.$
19. $4 - 5n - 6n^2.$
20. $39x^2 - 20xy - 11y^2.$

EXERCISE XLI.

Resolve into factors:

1. $6x^3 + 17xy + 12y^2 + 22xz + 31yz + 20z^2.$
2. $6x^2 + 19xy + 10y^2 + 13xz + 16yz + 6z^2.$
3. $12a^2 + 31ab + 20b^2 + 46ac + 59bc + 42c^2.$
4. $18a^2 + 42ab + 20b^2 + 48ac + 38bc + 14c^2.$
5. $18x^2 + 59xy + 35y^2 + 66x + 72y + 36.$
6. $6a^2 + 19ab + 15b^2 + 24a + 38b + 24.$
7. $2x^2 + 7xy + 3y^2 - 7xz - 11yz + 6z^2.$
8. $28x^2 - 43xy + 10y^2 + 5xz + 14yz - 12z^2.$
9. $10a^2 - 9ab + 2b^2 + 19ac - 7bc - 15c^2.$
10. $35x^2 - 26xy - 16y^2 - 13x + 38y - 12.$
11. $27a^2 - 3ab - 14b^2 - 87a + 85b - 44.$
12. $x^3 + y^3 + z^3 - 3xyz.$ 14. $x^3 - y^3 + z^3 + 3xyz.$
13. $x^3 + y^3 - z^3 + 3xyz.$ 15. $x^3 - y^3 - z^3 - 3xyz.$

FACTORS, MULTIPLES AND FRACTIONS.

EXERCISE XLII.

Find the highest common factor of:

A.

1. $x^2 + xy$; $x^2 - y^2$.
2. $(x+y)^2$; $x^2 - y^2$.
3. $2a^2 + 2ay$; $a^3 - a^2y$.
4. $4x^2 - 9y^2$; $6x^2 - 9xy$.
5. $a^3b^2 - a^2b^3$; $a^4b^3 - a^3b^4$.
6. $x^3 + y^3$; $x^3 + x^2y$.
7. $x^2 - 4y^2$; $x^2 + 2xy$.
8. $a^3 - a^2x$; $a^4 - ax^3$.
9. $a^2bx + ab^2x$; $a^2b - b^3$.
10. $c^2 - d^2$; $c^2d - cd^2$.
11. $xy - y$; $x^4y - xy$.
12. $a^4 - x^4$; $a^2x + x^3$.
13. $x^2 - 9x + 20$; $x^2 - x - 20$.
14. $x^2 + 2x - 120$; $x^2 - 2x - 80$.
15. $x^2 - 9x - 36$; $x^2 - 15x + 36$.
16. $x^2 - 9$; $x^2 - 18x + 45$.
17. $x^3 + 8y^3$; $x^2 + xy - 2y^2$.
18. $a^2 - 17a - 60$; $a^2 + 23a + 60$.
19. $x^2 + 7x + 10$; $x^2 + 6x + 8$.
20. $(x - 3y)^2$; $x^4 - 27xy^3$.
21. $15x^2 + 8x + 1$; $12x^2 + x - 1$.
22. $3x^2 - 7x - 6$; $2x^2 - 7x + 3$.
23. $a^2 - ab - 12b^2$; $a^2 + 5ab + 6b^2$.
24. $ax^2 + 2a^2x + a^3$; $3(ax + a^2)^2$.

B.

1. $x^3 - 6x^2 - 86x + 35$; $x^3 - 5x^2 - 99x + 40$.
2. $a^3 + 2a^2 - 13a + 10$; $a^3 + a^2 - 10a + 8$.
3. $2x^3 - 5x^2 + 11x + 7$; $4x^3 - 11x^2 + 25x + 7$.
4. $4x^3 - 3x^2 - 24x - 9$; $8x^3 - 2x^2 - 53x - 39$.
5. $a^3 - a^2 + a + 3$; $a^4 + a^3 - 3a^2 - a + 2$.
6. $7x^3 - 46x^2 - 22x + 7$; $49x^3 + 49x^2 + 5x - 4$.
7. $x^4 - 2x^3 + x^2 - 8x + 8$; $8x^3 - 24x^2 + 18x - 2$.
8. $4a^3 - 32a^2 + 85a - 75$; $3a^3 - 15a^2 + 15a + 9$.

9. $36x^3 - 6ax^2 + 36a^2x - 210a^3$; $6x^3 + 14ax^2 - 94a^2x + 90a^3$.

10. $15x^3 - 14x^2y + 24xy^2 - 7y^3$; $27x^3 + 33x^2y - 20xy^2 + 2y^3$.

11. $x^4 + x^2y^2 + y^4$; $x^4 + 2x^3y + 3x^2y^2 + 2xy^3 + y^4$.

12. $x^4 + 4x^2 + 16$; $x^5 + x^4 - 2x^3 + 17x^2 - 10x + 20$.

EXERCISE XLIII.

Reduce to lowest terms:

A.

1. $\dfrac{ax}{a^2x^2 - ax}$.

2. $\dfrac{10(x^3 - y^3)}{5x^2 + 5xy + 5y^2}$.

3. $\dfrac{4a^2 - c^2}{4a^2 + 4ac + c^2}$.

4. $\dfrac{5x^3y + 10x^2y^2}{3x^2y^2 + 6xy^3}$.

5. $\dfrac{10b^2 + 20bc + 10c^2}{5b^3 + 5b^2c}$.

6. $\dfrac{x^3 - 1}{x^3 - 3x^2 + 3x - 1}$.

7. $\dfrac{x^4 + a^2x^2 + a^4}{x^4 + ax^3 - a^3x - a^4}$.

8. $\dfrac{abx + bx^2}{adx + dx^2}$.

9. $\dfrac{x^3 - 2xz^2}{x^4 - 4x^2z^2 + 4z^4}$.

10. $\dfrac{x^2 - 5x}{x^2 - 4x - 5}$.

12. $\dfrac{3x^2 + 26x + 35}{2x^2 + 17x + 21}$.

13. $\dfrac{(a+b)^2 - c^2}{(a+b+c)^2}$.

14. $\dfrac{20x^4 + x^2 - 1}{25x^4 + 5x^2 - x - 1}$.

15. $\dfrac{4a^2 - 9b^2}{4a^2 + 6ab}$.

16. $\dfrac{ay + y^2}{abc + bcy}$.

17. $\dfrac{3a^2 + 6a}{a^2 + 4a + 4}$.

18. $\dfrac{xy - xyz}{3az - 3az^2}$.

19. $\dfrac{x^3y + 2x^2y + 4xy}{x^2 - 8}$.

20. $\dfrac{1 - x^2}{1 + x + y + xy}$.

21. $\dfrac{27x + x^4}{18x - 6x^2 + 2x^3}$.

11. $\dfrac{3mx + 5nx^2}{6my + 10nxy}$.

22. $\dfrac{x^3 + 10x^2 + 29x + 20}{x^3 + 11x^2 + 38x + 40}$.

B.

1. $\dfrac{x^3 + 2x^2 - 3x + 20}{x^3 - 4x^2 + 9x - 10}$.

2. $\dfrac{x^3 - x^2 + 3x + 5}{x^3 - 5x^2 + 11x - 15}$.

3. $\dfrac{x^4 - x^3y - xy^3 - y^4}{x^4 + x^3y + xy^3 - y^4}$.

4. $\dfrac{6x^3 - 17x^2 + 11x - 2}{6x^3 - 23x^2 + 16x - 3}$.

5. $\dfrac{x^3 - 2x^2 - x + 2}{x^3 - 6x^2 + 11x - 6}$.

6. $\dfrac{m^3 + 3m^2 + 5m + 3}{m^3 + m^2 + m - 3}$.

7. $\dfrac{x^4 + 4x^2 + 16}{x^4 - x^3 + 8x - 8}$.

8. $\dfrac{x^4 - 8ax^3 + 33a^2x^2 - 76a^3x + 77a^4}{x^5 - 5ax^4 + 2a^2x^3 + 32a^3x^2 - 70a^4x + 49a^5}$.

9. $\dfrac{x^4 + 4x^3 - 5x + 2}{x^5 - x^4 - 3x^3 + 5x^2 + 3x - 3}$.

10. $\dfrac{3x^4 + 4x^3 - 6x^2 - 12x - 5}{x^5 - 2x^4 - 6x^3 + 4x^2 + 13x + 6}$.

11. $\dfrac{15x^3 - 38x^2 - 2x + 21}{3x^3 - 13x^2 + 23x - 21}$.

12. $\dfrac{a^3 - a^2b - ab^2 - 2b^3}{a^3 + 3a^2b + 3ab^2 + 2b^3}$.

13. $\dfrac{a^3 + a^2 - 10a + 8}{a^3 + 2a^2 - 13a + 10}$.

14. $\dfrac{2 + 4x^2 + 2x^3 + 4x^4}{3 + 9x^2 + 6x^3 + 9x^4}$.

15. $\dfrac{3x^3 - 27ax^2 + 78a^2x - 72a^3}{6x^3 + 30ax^2 - 12a^2x - 144a^3}$.

16. $\dfrac{3x^3 - 13x^2 + 23x - 21}{15x^3 - 38x^2 - 2x + 21}$.

EXERCISE XLIV.

Simplify:

1. $\dfrac{x-y}{x^2+xy} \times \dfrac{x^2-y^2}{x^2-xy}$.

4. $\dfrac{25a^2-b^2}{9a^2x^2-4x^2} \times \dfrac{3ax+2x}{5a+b}$.

2. $\dfrac{a^2+4a}{a^2-3a} \times \dfrac{4a^2-12a}{3a^2+12a}$.

5. $\dfrac{x+11}{x+3} \div \dfrac{x^2-121}{x^2-9}$.

3. $\dfrac{16x^2-9y^2}{x^2-4} \times \dfrac{x-2}{4x-3y}$.

6. $\dfrac{2a+1}{ab+3} \div \dfrac{4a^2-1}{a^2b^2+3ab}$.

7. $\dfrac{x^2-7x}{x^2+x-2} \times \dfrac{x^2+2x}{x^2-13x+42}$.

8. $\dfrac{x^2-5x}{x^2-3x} \div \dfrac{x^2-11x+30}{x^2-6x+9}$.

9. $\dfrac{x^2+2x}{x^2-25} \div \dfrac{x^2-4}{x^2+5x}$.

10. $\dfrac{x^2-6x-27}{x^2-12x-45} \div \dfrac{x^2-4x-45}{x^2-14x-15}$.

11. $\dfrac{x^2+4x}{2x^2+9x+4} \times \dfrac{2x^2+5x+2}{x^2-4}$.

12. $\dfrac{x+3}{x+4} \div (x^2+7x+12)$.

13. $\dfrac{x-5}{x-6} \times (x^2-11x+30)$.

14. $\dfrac{x^2-y^2}{(x+y)^2} \div \dfrac{x-y}{x+y}$.

15. $\dfrac{(a-b)^2}{a^2} \div \dfrac{1}{(a-b)^3}$.

16. $\dfrac{(a+b)^2-c^2}{a^2-(b-c)^2} \times \dfrac{c^2-(a-b)^2}{c^2-(a+b)^2}$.

17. $\dfrac{x^2+2xy+y^2-z^2}{x^2-2xy+y^2-z^2} \times \dfrac{x-y+z}{x+y-z}$.

18. $\dfrac{x^2-(m-n)^2}{x^2-(n-m)^2} \times \dfrac{(x-n)^2-m^2}{(x-m)^2-n^2}.$

19. $\dfrac{x^2-x-2}{x^2+2x-8} \times \dfrac{x^2-x-20}{x^2-25} \div \dfrac{x+1}{x^2+5x}.$

20. $\dfrac{x^2-8x-9}{x^2-17x+72} \div \dfrac{x^2+4x-5}{x^2-9x+8} \times \dfrac{x^2-25}{x^2-1}.$

21. $\dfrac{x^3-y^3}{x^3+y^3} \times \dfrac{x+y}{(x-y)^3} \times \dfrac{(x-y)^2+xy}{(x+y)^2-xy}.$

22. $\dfrac{(x+a)^2(x-b)^2}{(x+b)^2(x-c)^3} \div \dfrac{(x+a)^3(x-b)^3}{(x+b)\;(x-c)^2}.$

23. $\dfrac{x^6-y^6}{x^4+2x^2y^2+y^4} \times \dfrac{x^2+y^2}{x^2-xy+y^2} \times \dfrac{x+y}{x^3-y^3}.$

24. $\dfrac{4x^2-16x+15}{2x^2+3x+1} \div \dfrac{2x^2-17x+21}{x^2-6x-7} \times \dfrac{4x^2-1}{4x^2-20x+25}.$

25. $\dfrac{(a-b)^3}{m^3+n^3} \div \left(\dfrac{a^2-2ab+b^2}{m+n} \times \dfrac{a^2-b^2}{m^2-mn+n^2}\right).$

26. $\dfrac{x^2-7x+12}{x^2-x} \times \dfrac{x^3-1}{x^2-4x} \div \dfrac{x^2+x+1}{x^2}.$

EXERCISE XLV.

Find the least common multiple of:

1. x^2-1 ; $x^2+x.$
2. a^2+ab ; $ab+b^2.$
3. $2x^2+x$; $4x^2y-y.$
4. $6x^2-2x$; $9x^2-3x.$
5. $a+b$; $a^3+b^3.$
6. $4a^2b-b$; $2a^2+a.$
7. x^2-4 ; $x^2+2x.$
8. $(x-1)^2$; $x^2-1.$
9. x^2+4x+4 ; $x^2+5x+6.$
10. x^2-6x+8 ; $x^2-5x+4.$
11. $x^2-11x+30$; $x^2+2x-35.$
12. $x^2+8x+15$; $x^2+9x+20.$
13. $x^2-9x-22$; $x^2-13x+22.$
14. $2x^2+3x+1$; $x^2-x-2.$
15. x^3-27 ; $x^2-15x+36.$

16. $(x+4)^3$; x^3+64.

17. x^2+x-20 ; $x^2-10x+24$; x^2-x-30.

18. x^2+3x+2 ; $2x^2+3x+1$; $2x^2+5x+2$.

19. x^2+5x+6 ; $5x^2+11x+2$; $5x^2+16x+3$.

20. x^2-4 ; $3x^2-x-14$; $3x^2-13x+14$.

21. $12x^2+3x-42$; $12x^2+30x+12$; $16x^2-20x-14$.

22. $2x^2-5xy-7y^2$; $4x^2-xy-5y^2$; $8x^2-38xy+35y^2$.

23. $a^3+2a^2b-ab^2-2b^3$; $a^3-2a^2b-ab^2+2b^3$.

24. $x^3+13x^2+56x+80$; $x^3+12x^2+47x+60$.

EXERCISE XLVI.

Find the value of:

A.

1. $\dfrac{4x+7}{5}+\dfrac{3x+4}{15}$.

6. $\dfrac{2x+5}{x}-\dfrac{x+3}{2x}-\dfrac{17}{8x^2}$.

2. $\dfrac{x+1}{2}+\dfrac{x+3}{5}+\dfrac{x+7}{10}$.

7. $\dfrac{x+3}{17x}-\dfrac{x-5}{34x}+\dfrac{x-2}{51x}$.

3. $\dfrac{2x-1}{3}+\dfrac{3x-2}{4}+\dfrac{4x-3}{6}$.

8. $\dfrac{a-2b}{2a}-\dfrac{a-4b}{4a}+\dfrac{a-8b}{8a}$.

4. $\dfrac{5x+8}{12}+\dfrac{2x-3}{9}-\dfrac{x+2}{6}$.

9. $\dfrac{a-b}{ab}+\dfrac{b-c}{bc}+\dfrac{c-a}{ac}$.

5. $\dfrac{2x-14}{30}+\dfrac{x-9}{25}-\dfrac{x+3}{45}$.

10. $\dfrac{4x+5}{3}-\dfrac{3x-7}{5x}+\dfrac{9}{12x^2}$.

11. $\dfrac{3x-2y}{xy}+\dfrac{4x-3z}{xz}+\dfrac{5}{x}$.

12. $\dfrac{a^2-bc}{bc}-\dfrac{b^2-ac}{ac}-\dfrac{c^2-ab}{ab}$.

13. $\dfrac{5a+2b}{3c}-\dfrac{4c-3b}{ac^2}-\dfrac{5ac-2c^2}{a^2c^2}$.

14. $\dfrac{x-3}{5x}+\dfrac{2x^2-18}{20x^2}-\dfrac{8-x^3}{15x^3}$.

15. $\dfrac{x-2y}{xy} + \dfrac{3y-a}{ay} + \dfrac{3x-2a}{ax}$.

16. $\dfrac{1}{x+2} + \dfrac{1}{x+3}$.

17. $\dfrac{3}{x+4} + \dfrac{4}{x+5}$.

18. $\dfrac{4}{x-5} - \dfrac{3}{x-4}$.

19. $\dfrac{3}{x-6} - \dfrac{1}{x+2}$.

20. $\dfrac{a}{x-a} + \dfrac{b}{x-b}$.

21. $\dfrac{a}{x+a} - \dfrac{b}{x+b}$.

22. $\dfrac{x+y}{x-y} - \dfrac{x-y}{x+y}$.

23. $\dfrac{x}{x+y} + \dfrac{x}{x-y}$.

24. $\dfrac{x-4}{x-3} - \dfrac{x-6}{x-5}$.

25. $\dfrac{x+5}{x-5} - \dfrac{x-5}{x+5}$.

26. $\dfrac{x}{1-x^2} - \dfrac{x}{1+x^2}$.

27. $\dfrac{2a^2}{a^2-b^2} - \dfrac{2a}{a+b}$.

28. $\dfrac{x^2+xy+y^2}{x+y} + \dfrac{x^2-xy+y^2}{x-y}$.

29. $\dfrac{a^3+b^3}{a^2-ab+b^2} - \dfrac{a^3-b^3}{a^2+ab+b^2}$.

30. $\dfrac{1}{x(x-y)} + \dfrac{1}{y(x+y)}$.

B.

1. $\dfrac{1}{x+y} - \dfrac{1}{x-y} + \dfrac{2x}{x^2-y^2}$.

2. $\dfrac{2}{a+x} + \dfrac{2}{a-x} - \dfrac{2a}{a^2-x^2}$.

3. $\dfrac{a+x}{a-x} + \dfrac{a-x}{a+x} - \dfrac{a^2-x^2}{a^2+x^2}$.

4. $\dfrac{2}{x+1} + \dfrac{3}{x+2} + \dfrac{4}{x+3}$.

5. $\dfrac{x^2}{x^2-1} + \dfrac{x}{x-1} + \dfrac{x}{x+1}$.

6. $\dfrac{2a}{2a+3b} + \dfrac{3b}{2a-3b} - \dfrac{8b^2}{4a^2-9b^2}$.

7. $\dfrac{4}{x-2} + \dfrac{5}{3x+6} + \dfrac{7x}{x^2-4}$.

8. $\dfrac{6}{x^2-9x+20} + \dfrac{5}{x^2-11x+30}$.

9. $\dfrac{x-5}{x^2-7x+12} - \dfrac{x-7}{x^2-5x+6}$.

10. $\dfrac{a+b}{(b-c)(c-a)} + \dfrac{b+c}{(c-a)(a-b)} + \dfrac{c+a}{(a-b)(b-c)}$.

11. $\dfrac{x^2+y^2}{xy} - \dfrac{x^2}{xy+y^2} - \dfrac{y^2}{x^2+xy}$.

12. $\dfrac{1}{x-y} + \dfrac{x-y}{x^2+xy+y^2} + \dfrac{xy-2x^2}{x^3-y^3}$.

13. $\dfrac{10}{x^2+5x+6} + \dfrac{12}{x^2+9x+14} - \dfrac{15}{x^2+10x+21}$.

14. $\dfrac{a+c}{(a-b)(x-a)} - \dfrac{b+c}{(a-b)(x-b)}$.

15. $\dfrac{a}{(a-b)(a-c)} + \dfrac{b}{(b-a)(b-c)} + \dfrac{c}{(c-a)(c-b)}$.

16. $\dfrac{a^2-bc}{(a+b)(a+c)} + \dfrac{b^2-ac}{(b+a)(b+c)} + \dfrac{c^2-ab}{(c+a)(c+b)}$.

17. $\dfrac{a+b}{(a-b)(a-c)} + \dfrac{2b}{(b-a)(b-c)} + \dfrac{b+c}{(c-a)(c-b)}$.

18. $\dfrac{1+x}{1+x+x^2} + \dfrac{1-x}{1-x+x^2} - \dfrac{2}{1+x^2+x^4}$.

19. $\left(\dfrac{a+b}{a-b} + \dfrac{a-b}{a+b}\right) \div \left(\dfrac{a+b}{a-b} - \dfrac{a-b}{a+b}\right)$

20. $\dfrac{1}{x-3a} - \dfrac{1}{x+3a} + \dfrac{3}{x+a} - \dfrac{3}{x-a}$.

FRACTIONAL EQUATIONS.

EXERCISE XLVII.

Solve:

1. $\dfrac{x}{2} + \dfrac{2x - 10}{3} = 5.$

2. $\dfrac{x - 5}{10} + \dfrac{x + 5}{5} = 5.$

3. $\dfrac{3x - 6}{2} + \dfrac{x + 10}{3} = 15.$

4. $\dfrac{x + 19}{5} - \dfrac{x}{4} = 3.$

5. $\dfrac{x - 4}{14} = \dfrac{x - 10}{10}.$

6. $\dfrac{3x - 3}{8} = 3 + \dfrac{x + 1}{6}.$

7. $\dfrac{8(x + 2)}{5} = 14 + \dfrac{10x}{13}.$

8. $\dfrac{x + 4}{7} + \dfrac{x - 4}{3} = 1.$

9. $\dfrac{x - 8}{7} + \dfrac{5}{21} = \dfrac{3 - x}{3}.$

10. $\dfrac{x + 5}{6} - \dfrac{x + 3}{4} = \dfrac{x + 1}{9}.$

11. $\dfrac{2x}{3} + 5 = \dfrac{7x}{12} + 10.$

12. $\dfrac{x + 2}{10} + \dfrac{x - 1}{14} = \dfrac{x - 2}{4}.$

13. $\dfrac{2x}{3} + 13 = \dfrac{4x}{5} + 7.$

14. $\dfrac{x + 9}{2} + \dfrac{4x}{7} = \dfrac{6x - 12}{5} + 6.$

15. $\dfrac{7x}{8} - 4 = \dfrac{9x}{10} - 7.$

16. $\dfrac{29 - 11x}{3} + \dfrac{28x + 14}{21} = \dfrac{17 - 3x}{5}.$

17. $\dfrac{x}{4} + \dfrac{x}{6} + \dfrac{x}{8} = \dfrac{13}{24}.$

18. $\dfrac{9x}{4} + \dfrac{3 - x}{2} = \dfrac{29x}{8} - 43\tfrac{1}{2}.$

19. $\dfrac{x + 5}{7} + \dfrac{x - 2}{3} = \dfrac{x + 2}{4}.$

20. $\dfrac{17 - 2x}{5x} - \dfrac{4x + 2}{3x} = 1.$

21. $x - \dfrac{x-2}{3} = \dfrac{x+23}{4} - \dfrac{10+x}{5}$.

22. $\dfrac{x-9}{8} - \dfrac{3}{7}(55-x)+4 = \dfrac{x+15}{28}$.

23. $\dfrac{2}{3}(x-5) - \dfrac{3}{11}\left(x - 13\dfrac{1}{3}\right) = 15 - \dfrac{3}{5}\left(19 - \dfrac{x}{3}\right)$.

24. $2x - 1 + \dfrac{7x-2}{3} - \dfrac{3x+4}{5} = \dfrac{7x-4}{5} + \dfrac{5x+1}{3}$.

25. $.5x+.6x-.8 = .75x+.25$.

26. $\dfrac{3x+2}{x-3} - \dfrac{3x-2}{x+3} = \dfrac{4x+36}{x^2-9}$.

27. $\dfrac{x-4}{x-5} - \dfrac{x-5}{x-6} = \dfrac{x-7}{x-8} - \dfrac{x-8}{x-9}$.

28. $x + \dfrac{3x-5}{2} = 12 - \dfrac{2x-4}{3}$.

29. $3 + \dfrac{x}{4} = \dfrac{1}{2}\left(4 - \dfrac{x}{3}\right) - \dfrac{5}{6} + \dfrac{1}{3}\left(11 - \dfrac{x}{2}\right)$.

30. $7 - \dfrac{23-x}{5} = \dfrac{1}{5}(x-8) + \dfrac{4+x}{4} + \dfrac{x-1}{7}$.

PROBLEMS LEADING TO FRAC-
TIONAL EQUATIONS.

EXERCISE XLVIII.

A.

1. Find a number such that the sum of its sixth and ninth parts may be equal to 20.

2. What is the number whose eighth, sixth and fourth parts together make up 26?

3. What is the number of which the twelfth, twentieth and fortieth parts added together give as a result 76?

4. There is a number whose fifth part is less than its fourth part by 36; find it.

5. Find a number such that six-sevenths of it shall exceed four-fifths of it by 18.

6. Two consecutive numbers are such that one-fifth of the greater exceeds one-seventh of the less by 3; find the numbers.

7. Two numbers differ by 16, and one is eight-ninths of the other; find them.

8. Find three consecutive numbers, such that if they be divided by 14, 9 and 20 respectively, the sum of the quotients will be 23.

9. Four-fifths of A's money is equal to B's, and two-thirds of B's is equal to C's; in all they have $595; what sum has each?

10. To a certain number I add its half, and the result is as much above 527 as the number itself is below 563. Find the number.

11. The width of a room is three-fourths of its length. If the width had been 3 feet more, and the length 3 feet less, the room would have been square; find its dimensions.

12. Divide $1,359 between A and B, so that B's share may be seven-eighths of A's share.

13. Find a number such that one-half, one-third, and one-fourth of it added together shall exceed the number itself by 4½.

14. Divide the number 112 into two parts, such that if 21 be added to the less, the sum shall be less than one-third of the greater by the third part of unity.

15. What sum is that from which, if $46.20 be subtracted, one-half of the remainder shall exceed one-third of the remainder by $50?

B.

1. A certain sum consists of two digits, such that the right hand digit exceeds the left hand digit by 2; and,

if the sum of the digits be increased by $\frac{2}{7}$ of the number, the digits will be inverted; required the number.

2. Divide the number 360 into four parts, such that the first increased by 2, the second diminished by 2, the third divided by 2, and the fourth multiplied by 2, shall all equal the same quantity.

3. Find a number, such that if 21 be taken from it, and the remainder divided by $8\frac{2}{3}$, the quotient will be 5.

4. What number is that to which, if 11 be added, two and a-half times the sum shall be 85?

5. What is the height of a house wall in which a window 6 feet high has under it $\frac{1}{3}$, and above it $\frac{1}{2}$ of the whole height?

6. A number consists of two digits, the first of which is greater than the second by unity, and the sum of the digits is one-sixth of the number itself; find it.

7. A, who walks at the rate of $3\frac{3}{4}$ miles per hour, starts 18 minutes before B; at what rate per hour must B walk to overtake A at the ninth mile-stone?

8. A, who travels $3\frac{1}{4}$ miles per hour, starts $2\frac{1}{2}$ hours before B, who is going at $4\frac{1}{2}$ miles an hour in the same direction. When will B overtake A?

9. How many minutes does it want to 4 o'clock, if three-quarters of an hour ago it was twice as many minutes past two o'clock?

10. The sum of $1,650 is laid out in two investments, by one of which 15 per cent. is gained, and by the other 8 per cent. is lost, and the amount of the returns is $1,725. Find each investment.

11. A person goes from Hamilton to Toronto by boat at the rate of 13 miles an hour, remains an hour and a half in Toronto, and returns by rail at the rate of twenty-six miles an hour. He is gone altogether six hours; find the distance from Hamilton to Toronto.

12. The sum of two numbers is one-fourth of their product, and if 6 be divided by the first number and 3 by the second the sum of the quotients is 1; find the numbers.

SIMULTANEOUS EQUATIONS.

EXERCISE XLIX.

A.

1. $3x + 7y = 27$.
$5x + 2y = 16$.

2. $7x + 2y = 47$.
$5x - 4y = 1$

3. $5x + 4y = 58$.
$6x + 14y = 134$

4. $5x + 8y = 101$.
$9x + 2y = 95$.

5. $6x + 35y = 177$.
$8x - 21y = 33$

6. $2x + 7y = 52$.
$3x - 5y = 16$

7. $4x + 9y = 79$.
$7x - 17y = 40$

8. $15x - 13y = 78$.
$7x - 4y = 55$

9. $13x - 2y = 57$.
$5x + 9y = 188$.

10. $72x + 14y = 330$.
$63x + 7y = 273$

B.

1. $\dfrac{x}{2} + \dfrac{y}{3} = 7.$

$\dfrac{x}{3} + \dfrac{y}{2} = 8.$

2. $\dfrac{2x}{3} + y = 16.$

$x + \dfrac{y}{4} = 14.$

3. $\dfrac{5}{6}x - y = 3$

$x - \dfrac{5}{6}y = 8.$

4. $\dfrac{1}{2}x - \dfrac{1}{5}y = 4.$

$\dfrac{1}{7}x + \dfrac{1}{15}y = 3.$

5. $\dfrac{x}{7} + \dfrac{y}{5} = 1\dfrac{3}{7}.$

$x + \dfrac{y}{3} = 4\dfrac{2}{3}.$

6. $\dfrac{x+y}{3} + 5 = 10.$

$\dfrac{x-y}{2} + 7 = 9\dfrac{1}{2}.$

7. $\dfrac{2x}{7} + \dfrac{5y}{3} = 7.$

$3x - 7y = 0.$

8. $2x - \dfrac{y-3}{5} = 4.$

$3y = 9 - \dfrac{x-2}{3}.$

9.
$$2x - \frac{y+3}{4} = 7 + \frac{3y - 2x}{5}.$$
$$4y - \frac{8-x}{3} = 24\frac{1}{2} - \frac{2y+1}{2}.$$

10.
$$\frac{1}{x} + \frac{1}{y} = a.$$
$$\frac{1}{x} - \frac{1}{y} = b.$$

EXERCISE L.

Problems:

1. The cost of 7 lbs. of tea and 5 lb. of coffee is $5.15; the cost of 4 lbs. of tea and 9 lbs. of coffee is $5.40. What is the cost of 1 lb. of each?

2. Six pounds of tea and eleven pounds of sugar cost $3.54, and eleven pounds of tea and six pounds of sugar cost $5.64. Find the cost of tea and sugar per pound.

3. Five turkeys and four geese can be bought for $5.76, and seven turkeys and three geese can be bought for $6.66. What is the value of each fowl?

4. Thirteen horses and eight cows can be bought for $1,166, and nine horses and twelve cows can be bought for $1,014. What is the value of each animal?

5. If the numerator of a fraction be increased by 2 and the denominator by 4 it becomes equal to $\frac{13}{21}$; and, if the numerator and denominator are each diminished by 3, it becomes equal to $\frac{4}{7}$. Find the fraction.

6. Three times B's age exceeds A's age by 72 years, and one-half of A's age is less than B's age by 17 years. Find their ages.

7. The sum and the difference of a number of two digits and of the number formed by reversing the digits are 143 and 45 respectively. Find the number.

8.　The wages of nine men and eight boys amount to $133.20 for the week ; if 4 men together receive $16.20 more than 6 boys, what is the wages of each man and boy for the week?

9.　In 13 hours A walks 13½ miles more than B does in 12 hours, and in 9 hours B walks 6¾ miles more than A does in six hours.　How many miles does each walk per hour?

10.　A farmer bought 100 acres of land for $4,122, part at $37.50 an acre and the remainder at $45.25 an acre.　How many acres had he of each kind ?

MISCELLANEOUS EXAMPLES.

A.

1.　Simplify $2b - \{ b - (a + b) - [b - (b - \overline{a - b})] + 2a \}$.

2.　Find the sum of $a + b - 3 (c + d)$,　$b + c - 4 (d + a)$, and $c + d - 5 (a + b)$.

3.　If $x = 8$, $y = 7$, $z = 6$, find the value of $\sqrt[3]{3x + 4y + 2z}$.

4.　Find the square of $2 - 3x + 4x^2 - 5x^3$.

5.　Subtract $2x^2 - 3y^2 - 4z^2$ from the sum of $5x^2 + 3y^2$, $4y^2 - 5z^2$, and $6z^2 - 7x^2$.

6.　Solve $\frac{3}{10} (2x - 7) - \frac{1}{3} (x - 8) = \frac{4x + 1}{30} + 2$.

7.　Find the H.C.F. and L.C.M. of $x^3 + 3x^2 - 4$ and $x^4 + 2x^2 - 2x + 4$.

8.　Simplify $\left(\frac{a}{x + a} - \frac{x}{x - a} \right) \times \frac{x^2 + ax}{x^2 + a^2}$.

9.　Show that the sum of $2a + 2b + 12c$, $24a + 12b - 2c$, and $-14a - 2b + 2c$ is twelve times the sum of $25a + 13b - 8c$, $-13a - 13b - c$, and $-11a + b + 10c$.

10.　Find the factors of (a) $10a^2 + 79a - 8$, (b) $729a^6 - b^6$.

11.　Solve $\frac{5x + 3}{17} + \frac{4x - 118}{11} = 3 - \frac{2x - 1}{5}$.

12. If $a = 1$, $b = -2$, $c = 3$, $d = -4$, find the value of $\sqrt{a^2 - 4b + d^2} - \sqrt{a + b^3 + c^3 + d}$.

13. Find an expression which will divide both $4x^3 + 7x^2 - 3x - 15$ and $8x^2 + 5x - 9$ without remainder.

14. Divide $3 - 15x + \dfrac{304}{15}x^2 + 10x^3 - \dfrac{1244}{75}x^4 - \dfrac{112}{15}x^5$
$$\text{by } 1 - x - \frac{14}{15}x^2.$$

15. Simplify $\dfrac{49a^2b^2c^2 - 35b^3c^3}{91a^5bc - 65a^3b^2c^2}$.

16. Subtract $\dfrac{x+3}{x^2+x-12}$ from $\dfrac{x+4}{x^2-x-12}$.

17. Find the H.C.F. of $ax^2 + ab - a^2 - bx^2$ and $a^2b + b^2c - abc - ab^2$.

18. Find the factors of $x^2 + 3xy - 40y^2 + x - 5y$.

19. Resolve $4(a^2x^3 - 8a^5) - 9(b^2x^3 - 8a^3b^2)$ into four factors; also $m^6 - 64$.

20. Find the value of
$$\frac{1}{(a-b)(b-c)} - \frac{1}{(b-c)(a-c)} - \frac{1}{(c-a)(b-a)}.$$

21. Divide $a^4b^2 + b^4c^2 + c^4a^2 - a^2b^4 - b^2c^4 - c^2a^4$ by $a^2b + b^2c + c^2a - ab^2 - bc^2 - ca^2$.

22. Simplify
$$\frac{a^2}{(b-a)(c-a)} + \frac{b^2}{(c-b)(a-b)} + \frac{c^2}{(a-c)(b-c)}.$$

23. Find the value of $\dfrac{7 + 3\sqrt{5}}{7 - 3\sqrt{5}} + \dfrac{7 - 3\sqrt{5}}{7 + 3\sqrt{5}}$.

24. Find the remainder when $x^4 - 2x^3 + x - 7$ is divided by $x + 2$; also find the value of this expression when $x = -2$.

25. What number must be added to $x^3 + 2x^2$ in order that the expression may be divisible by $x + 4$?

26. If $2s = a + b + c$, show that
$$(s-a)^2 + (s-b)^2 + (s-c)^2 + s^2 = a^2 + b^2 + c^2.$$

27. Find the continued product of $x+2$, x^2+2x+4, $x-2$, x^2-2x+4.

28. Divide the product of $x^2+8x+12$ and $x+4$ by x^2+6x+8.

29. Divide the product of $6a^2+23a+20$ and $20a^2-47a+21$ by $8a^2+6a-35$.

30. Multiply $(a^2+a+1)x-a-1$ by $(a-1)x-a^2+a-1$.

31. If $x+y=2a$ and $x-y=2b$, find the value of $x^4-2x^2y^2+y^4$.

32. Find the value of $x^4-2x^3y+2xy^3-y^4$ when $x=a+b$ and $y=a-b$.

33. Resolve $a^{16}-b^{16}$ into five factors.

34. Solve $\dfrac{3x-2}{x+5}=\dfrac{6x-3}{2x+7}$.

35. If $x+y=29$ and $x-y=21$, find the value of $4xy$.

36. If $a+b+c=105$, find the value of $a(a+b)+b(b+c)+c(c+a)+ab+ac+bc$.

37. Find the H.C.F. of $2x^3-7x^2-15x+27$ and $2x^2-13x+18$.

38. Simplify $\dfrac{(y+z-2x)^2-(z+x-2y)^2}{(x+y+z)^2-(x+y-5z)^2}$.

39. Solve $\dfrac{3x-5\frac{1}{2}}{2}+\dfrac{2x-1\frac{1}{2}}{3}=\dfrac{1}{4}\left(5\dfrac{1}{3}-\dfrac{x}{2}\right)$.

40. Employ factors to find the result of dividing $a^4+b^2c^2-a^2c^2-a^2b^2$ by $a^2+ac-bc-ab$.

41. Find the H.C.F. of $7x^4-10x^3y+3x^2y^2-4xy^3+4y^4$ and $8x^4-13x^3y+5x^2y^2-3xy^3+3y^4$.

42. What are the factors of $x^2-88x+1612$?

43. Divide $k^2l^3-m(k^2+m)l+km^2$ by $kl-m$, and $\dfrac{3}{4}a^5-4a^4+\dfrac{77}{8}a^3-\dfrac{43}{4}a^2-\dfrac{33}{4}a+27$ by $\dfrac{a^2}{2}-a+3$ without removing the bracket, or bringing the fractions to a common denominator.

44. Simplify
$$\left(m-\dfrac{m^2+n^2}{n}\right)\times\left(\dfrac{m^2-n^2}{m^3+n^3}\right)\div\left(\dfrac{1}{m}-\dfrac{1}{n}\right).$$

45. Divide the product of $a-b+c$, $a+b-c$, and $b+c-a$ by $a^2-b^2-c^2+2bc$.

46. Factor $6x^3-4x^2y-3xy^2+2y^3$.

47. Break up $\dfrac{3x^3+2x^2-x+1}{xy}$ into simple fractions.

48. If $x=3a-4b+5c$, $y=5b-4a-3c$, $z=4c-5a+2b$, find the value of $x+y+z$.

49. If $x=7m+8n+9p$, $y=5m-3n-2p$, $z=m-n+p$, find the value of $x-y+z$.

50. If $x=a+2b-3c$, $y=3a-4b+7c$, $z=5a+9b-11c$, find the value of $3x+4y+2z$.

51. If $x=17a-3b-12c$, $y=16b-4c-5a$, $z=15c-3a-2b$, find the value of $5x-7y-9z$.

52. Use the formula $(x+y)(x-y)=x^2-y^2$ to find the value of $(a+b+c)(a+b-c)(a-b+c)(c-a+b)$.

53. Simplify (1) $\dfrac{3(5+2x)}{xy}-\dfrac{3ax+by+15a}{axy}+\dfrac{b}{2a}$.

(2) $\dfrac{25(x^2-y^2)}{36(x-y)^2}\times\dfrac{12x(x-y)}{x^2+xy}\div\dfrac{5}{3x^2}$.

54. Divide $x^4-2x^2y^2+y^4-x^2-3y^2-2y$ by x^2-y^2-2y-1.

55. Find the factors of $x^2-60x+891$ and $a^2-4b^2+a+30b-56$.

56. Solve $\dfrac{x-1}{x-2}-\dfrac{x-2}{x-3}=\dfrac{x-4}{x-5}-\dfrac{x-5}{x-6}$.

57. Find the H. C. F. of $35x^3+47x^2+13x+1$ and $42x^4+41x^3-9x^2-9x-1$.

58. Solve $\frac{1}{27}(2x+7)-\frac{1}{15}(2x-7)=1\frac{5}{6}-\frac{1}{20}(3x+4)$.

59. Express in words :

x^n-y^n is divisible by $x-y$ always.

60. What number added to $4x^5+34x^4+58x^3+21x^2-123x-41$ will give a result divisible by $2x+13$?

61. Find the value of $10x^4-1109x^3-109x^2-212x-1111$ when $x=111$.

62. Find the H.C.F. of $4x^3-3x^2-24x-9$ and $8x^3-2x^2-53x-39$.

63. Solve $\dfrac{x-8}{x-10}+\dfrac{x-4}{x-6}=\dfrac{x-5}{x-7}+\dfrac{x-7}{x-9}$.

64. Find the factors of $x^3-6x^2+11x-6$.

65. If $x+y=m$, and $x-y=n$, express x^3+y^3 in terms of m and n.

66. Find the value of $x^4+x^2y^2+y^4$ when $x+y=2a$ and $x-y=2b$.

67. Find without actual multiplication the product of $2x^4-3x^3+4x^2-3x+2$ and $2x^4+3x^3+4x^2+3x+2$.

68. Simplify $(a^2+ab+b^2)^2-(a^2-ab+b^2)^2+(a^2+ab-b^2)^2-(a^2-ab-b^2)^2$.

69. Write down the continued product of $(2x+5)$ $(2x-7)(2x+9)(2x-11)$.

70. Simplify $5x^2+3xy-2y^2-(3x^2-5xy-7y^2)-(x^2-2xy+y^2)$ and find the value of the result when $x=2$ and $y=3$.

71. Find the value of $(x-y)^2+(y-z)^2+(z-x)^2$ when $x=2$, $y=3$, $z=4$.

72. Find the coefficient of x in the expansion of $(x-3)$ $(2x-4)(3x+7)$.

73. Form the square of x^3+2x^2-x+2.

74. Find the coefficient of x in the expansion of $(x-a)(x-2b)(x-3c)$.

75. Simplify $(a+b)^2(a-b)^2-(a^2+b^2)^2$.

76. Multiply $4a^2-3ab+7b^2$ by $3a^2-2ab-9b^2$, and prove the result by division.

77. Divide $8a^3-64b^3$ by $2a-4b$, and prove the result by multiplication.

78. Divide m^6-2m^3+1 by m^2-2m+1.

79. If $a=1$, $b=c=2$, $d=3$, find the value of $ab^2d-a^2cd^2+bcd-b^2(3ad-2b^2+4c)-b^3(4cd-3ac^2)$.

80. Solve the equation :

$(x-3)(x-5)+(x-6)(x-7)=(2x-9)(x-4)-23$.

81. Simplify $x^2-[(x-y)^2-\{(x-y-z)^2-(z-x)^2\}]$.

82. Find the H.C.F. of a^3b^2c, a^2bc^2, and abc^4 ; and also of $5x^3yz^2$, $12xy^3z$, and $20x^2y^2z^3$.

83. Find the H.C.F. of x^3+x^2+x+1 and x^3+3x^2+3x+1.

84. Simplify
$$\frac{(x-y)^2-z^2}{(x+y)^2-z^2} \times \frac{z^2-(x+y)^2}{(z-y)^2-x^2} \times \frac{x^2-(y-z)^2}{x^2-(y+z)^2} \times \frac{x+y+z}{x-y+z}.$$

85. Find the L. C. M. of $7(a-b)$, $14(a^2-b^2)$, and $21(a^3-b^3)$.

86. Simplify $\dfrac{x-1}{x+1} + \dfrac{1+x}{1-x} + \dfrac{4x^2}{x^2-1}$.

87. Simplify
$$\frac{2}{(x-2)(x-5)} + \frac{3}{(x-3)(5-x)} + \frac{1}{(3-x)(2-x)}.$$

88. Solve the equation :
$$x - \frac{x-2}{3} = 5\frac{3}{4} - \frac{x+10}{5} + \frac{x}{4}.$$

89. When $x=1$, $y=3$, $z=5$, find the value of
$$\frac{12x^3-y^2}{3x^2} + \frac{2z^2}{x+y^2} - \frac{x+y^2+z^3}{5y^3}.$$

90. Divide $x^4 + 10x^3 + 35x^2 + 50x + 24$ by $x+4$.

91. Simplify $25a - 19b - [3b - \{4a - (5b - 6c)\}] - 8a$.

92. Simplify
$$ad - [dc + d\{a - (c+b) - (d - \{c+b\} - c)\}].$$

93. Multiply $x^4 + 2ax^3 + 3a^2x^2 + 2a^3x + a^4$
$$\text{by } x^2 - 2ax + a^2.$$

94. Divide 1 by $1 - x + x^2$ to four terms.

95. Add together $a + 2x - y + 24b$, $3a - 4x - 2y - 81b$, $x + y - 22 + 55b$; and subtract the result from $3a + b + 3x + 2y$.

96. Simplify $\{x(x+a) - a(x-a)\} \{x(x-a) - a(a-x)\}$.

97. Multiply $3a^2 + ab - b^2$ by $a^2 - 2ab - 3b^2$, and divide the product by $a+b$.

98. Solve $x - \dfrac{x-2}{3} = \dfrac{x+23}{4} - \dfrac{10+x}{5}$.

99. If $x=1$, $y=-2$, $z=3$, find the value of
$$\frac{3x^2 - 2xy + 5y^2 + 5z^2 + 2yz + 2xz}{4x^2 + 2xy + 3y^2 + 2z^2 + yz - xz}.$$

100. Find the value of $x^6 - 102x^5 + 100x^4 + 102x^3 - 99x^2 - 201x$ when $x=101$.

B.

1. A certain number consists of two digits whose difference is 3, and, if the digits be inverted, the number so formed will be $\frac{4}{7}$ of the former. Find the original number.

2. Find a number such that if $\frac{3}{8}$ of it be subtracted from 20, and $\frac{5}{11}$ of the remainder from $\frac{1}{4}$ of the original number, 12 times the second remainder shall be half the original number.

3. A fish was caught whose tail weighed 9 lbs., his head weighed as much as his tail and half his body, and his body weighed as much as his head and tail. What did the fish weigh?

4. A can do a piece of work in 10 days; but after he has been upon it 4 days B is sent to help him, and they finish it together in 2 days. In what time would B have done the whole?

5. A cistern can be filled in half-an-hour by a pipe A, and emptied in 20 min. by another pipe B; after A has been opened 20 min. B is opened, and in 12 min. A is closed, and B remains open for 5 min. more. There are now 13 gallons in the cistern. How much would it hold when full?

6. Find the time between 2 and 3 o'clock at which the hour and minute hands of a watch are exactly opposite each other.

7. There are two bars of metal, the first containing 14 oz. of silver and 6 oz. of tin, the second containing 8 oz. of silver and 12 oz. of tin. How much must be taken from each to form a bar of 20 oz. containing equal weights of silver and tin?

8. A starts at 4 a.m. from X to walk to Y, a distance of 50 miles. B starts from X at 5 a.m., and, passing A at the twentieth milestone, reaches Y at 5 p.m. When will A arrive?

9. A man bought an equal number of two kinds of wine at 3 shillings and 4 shillings a bottle respectively. If he had spent his money equally he would have had

two more bottles than he had. How many bottles did he buy?

10. A starts from a certain place and travels at the rate of 7 miles in 5 hours; B starts from the same place 8 hours after A and travels in the same direction at the rate of 5 miles in 3 hours. How far will A have travelled when he is overtaken by B?

11. A general on attempting to draw up his army in the form of a solid square finds that he has 60 men over, and that he would require 41 men more in his army in order to increase the side of the square by one man, How many men are in the army?

12. A and B made a joint stock of $4,000 by which they gained $1,280, of which A had for his share $256 more than B. What did B contribute to the stock?

13. The tens digit of a number is 2 less than the units digit; and if the digits be inverted the new number is to the former as 7 to 4. Find the number.

14. A man can walk from P to Q and back again in a certain time at the rate of 4 miles an hour. If he walks at the rate of 3 miles an hour from P to Q, and at the rate of 5 miles an hour from Q to P, he requires 10 minutes longer for the double journey. What is the distance from P to Q?

15. The breadth of a room is two-thirds of its length. Had the breadth been 3 feet more and the length 3 feet less the room would have been square. Find the dimensions.

16. The sum of two numbers is one-fourth of their product, and if 6 be divided by the first number and 3 by the second the sum of the quotients is 1. Find the numbers.

17. A man goes into business with a certain capital which he finds has doubled itself by the end of the year. He then withdraws $1,000 to pay expenses and the remaining capital doubles itself during the second year; he then withdraws $1,000 as before, and so on for four years. He finds that he begins his fifth year with $5,000. How much had he to commence with?

18. The ingredients of a loaf of bread are rice, flour and water, and the loaf weighs 15 lbs. The weight of the rice increased by 5 lbs. is $\frac{3}{4}$ of the weight of the flour, and the weight of the water is $\frac{1}{5}$ of the weight of the flour and rice together. Find the weight of each.

19. A debtor is able to pay his creditors just 5s. in £1 ; but if his assets had been 5 times as great, and his debts $\frac{2}{3}$ of what they really were, he would have had a balance of £140. How much does he owe?

20. A boy selling oranges sells half his stock and one more to A, half of what remains and two more to B, and three that still remain to C. How many had he at first?

21. In a garrison of 2,744 men there are two cavalry soldiers to twenty-five infantry, and half as many artillery as cavalry. Find the numbers of each.

22. Divide 150 into two parts such that, if one be divided by 23 and the other by 27, the sum of the two quotients may be 6.

23. The first digit of a certain number exceeds the second by 4, and when the number is divided by the sum of the digits the quotient is 7. Find it.

24. The length of a floor exceeds the breadth by 4 feet ; if each had been increased by a foot the area of the room would have been increased by 27 square feet. Find the dimensions.

25. A person has travelled altogether 3,036 miles, of which he has gone 7 miles by water to 4 on foot, and 5 by water to 2 on horseback. How many did he travel each way?

26. I wish to enclose a piece of ground with palisades, and find that if I set them a foot asunder I shall have too few by 150, whereas, if I set them a yard asunder, I shall have too many by 70. What is the circuit of the piece of ground?

27. A man could reap a field by himself in 20 hours, but with his son's help for 6 hours he could do it in 16 hours. How long would the son be in reaping the field by himself?

28. Divide 144 into four such parts that the first increased by 3, the second diminished by 3, the third multiplied by 3 and the fourth divided by 3 shall all give the same result.

29. The sides of a rectangle are 12 and 20 feet. What is the breadth of the border which must be added all round that the whole area may be 384 square feet?

30. Find the price of eggs per dozen when two less in a shilling's worth raises the price one penny per dozen.

31. The difference between the squares of two consecutive numbers is 1503. Find the numbers.

32. What number is that, the double of which exceeds its half by 24?

33. A post is a fourth of its length in the mud, a third of its length in the water and 10 feet above the water. What is its length?

34. A is twice as old as B; 22 years ago he was three times as old. Required A's present age.

35. What sum of money is that from which, if $46.20 be subtracted, one-half of the remainder shall exceed one-third of the remainder by $50?

36. Divide 162 into three such parts that the first divided by 2, the second by 3, and the third by 4 shall give the same quotient.

37. Divide $2,481 among A, B and C so that B may have $72 more than A, and C $539 less than A and B together.

38. Divide $1,107 among A, B and C so that B may have half as much again as A, and C third as much again as B.

39. If 117 be added to a certain number the result is four times that number. Find the number.

40. Divide the number 132 into two parts such that five times one part may be equal to six times the other.

41. The sum of $745 was raised by A, B and C together; B contributed three times as much as A less $30, and C half as much as A and B together less $20. How much did each contribute?

42. A gentleman left $750 to be divided among four servants, of whom B was to have twice as much as A, C as much as A and B together, and D as much as A and C together. How much had each?

43. The sum of $18,259 was divided among four persons, so that the first and second together received $5801, the second and third together $8,023 and the third and fourth together $12,458. Find the share of each.

44. Find two consecutive numbers such that the fourth and the seventh of the first taken together shall be equal to the fifth and the sixth of the second taken together.

45. A herd cost $1,050, but, on 5 oxen being stolen, the rest average $10.50 a head more than at first. Find the number of oxen.

46. A person buys 8 lbs. of tea and 5 lbs. of sugar for $3.71, and at another time 6 lbs. of tea and 7 lbs. of sugar for $3.01. Find the price of tea and sugar per lb.

47. A farmer sold to one person 25 bushels of wheat and 30 bushels of oats for $27.20, to another person 35 bushels of wheat and 20 bushels of oats for $31.70. Find the price of wheat and oats per bushel.

48. Two trains, 92 feet long and 84 feet long respectively, are moving with uniform velocities on parallel rails; when they move in opposite directions they pass each other in $1\frac{1}{2}$ seconds, but when they move in the same direction the faster passes the other in 6 seconds. Find the rate of each train.

49. If one of two numbers be multiplied by 3 and the other by 4, the sum of the products is 43, and if the former be multiplied by 7 and the latter by 3, the difference between the results is 14. Find the numbers.

50. Five men and six women earn $116.10 in 6 days, and six men and ten women earn $108 in 4 days. Find the daily earnings of a man and a woman.

C.

Find the product of:

1. $3x^3 + 5x^2 + 7x + 9$ and $4x + 6$.
2. $4x^3 + 6x^2 + 8x + 10$ and $5x + 7$.

3. $5a^3 + 6a^2 + 7a + 8$ and $6a - 8$.
4. $6a^3 + 7a^2 + 8a + 9$ and $7a - 9$.
5. $2x^3 - 7x^2 + 3x - 6$ and $8x + 2$.
6. $6x^3 - 5x^2 + 4x - 7$ and $9x - 4$.
7. $9x^3 + 4x^2 + 7x + 6$ and $3x^2 + 4x + 5$.
8. $12a^3 - 10a^2 + 8a - 9$ and $4a^2 - 7a + 3$.
9. $11a^3 + 9a^2 - 7a - 5$ and $5a^2 + 6a - 7$.
10. $3x^2 + 7xy + 19y^2$ and $8x^2 + 5xy + 2y^2$.
11. $5a^2 - 12ab + 7b^2$ and $3a^2 - 4ab + 6b^2$.
12. $11x^2 - 5xy - 8y^2$ and $8x^2 + 5xy + 11y^2$.
13. $4x^4 + 5x^3 + 6x^2 + 7x + 8$ and $9x^2 + 10x + 11$.
14. $2x^4 - 3x^3 + 4x^2 - 5x + 7$ and $8x^2 - 9x + 10$.
15. $7a^4 - 6a^3b + 5a^2b^2 - 4ab^3 + 3b^4$ and $2a^2 - 4ab + 6b^2$.

Find the continued product of:

16. $2x + 3$, $3x + 4$, and $4x + 5$.
17. $5x - 6$, $6x - 7$, and $7x - 8$.
18. $3x + 4y$, $5x + 6y$, and $7x + 8y$.
19. $4a - 5b$, $6a - 7b$, and $8a - 9b$.
20. $6x^2 + 7x + 8$, $7x^2 + 8x + 9$, and $8x^2 + 9x + 10$.
21. $a^2 - 2ab + 3b^2$, $2a^2 - 3ab + 4b^2$, and $3a^2 - 4ab + 5b^2$.
22. $9x^2 - 8xy + 7y^2$, $6x^2 - 5xy + 4y^2$, and $3x^2 - 2xy + y^2$.
23. $11x^3 - 10x^2 + 9x - 8$, $7x^2 - 6x + 5$, and $4x - 3$.
24. $3x^2 + 4x + 5$, $2x + 3$, $3x^2 - 4x + 5$, and $2x - 3$.
25. $4a^2 - 5a + 7$, $3a - 5$, $4a^2 + 5a + 7$, and $3a + 5$.

Multiply together:

26. $a - x$, $a + x$, $a^2 + x^2$, and $a^4 + x^4$.
27. $x - 1$, $x - 3$, $x + 1$, and $x + 3$.
28. $x^2 + x + 1$, $x^2 - x + 1$, and $x^4 - x^2 + 1$.
29. The square of $x^2 - 3x + 2$ and $x^2 + 6x + 1$.
30. The square of $a + b$ and the cube of $a - b$.
31. $x^2 - xy + y^2 + x + y + 1$ and $x + y - 1$.
32. $a^2 + b^2 + c^2 + bc + ca - ab$ and $a + b - c$.
33. $15x^2 + 18ax - 14a^2$ and $4x^2 - 2ax - a^2$.
34. The square of $2x^2 - 3x + 4$ and $3x^2 - 5x + 6$.
35. The square of $3a^2 + 4a - 9$ and the square of $9a^2 - 4a + 3$.

D.

Find the quotient and remainder of:

1. $(14x^3 + 29x^2 + 22x + 25) \div (2x + 3)$.
2. $(24x^3 + 31x^2 + 31x + 19) \div (3x + 2)$.
3. $(35x^3 + 87x^2 + 94x + 57) \div (5x + 6)$.
4. $(28a^3 + 88a^2 + 89a + 26) \div (7a + 1)$.
5. $(20x^3 - x^2 - 2x - 30) \div (4x - 5)$.
6. $(12a^3 - 54a^2 - 6a + 32) \div (6a - 3)$.
7. $(36x^3 + 79x^2y - 44xy^2 - 13y^3) \div (9x + 4y)$.
8. $(72x^3 + 47x^2y + 34xy^2 + 65y^3) \div (8x + 7y)$.
9. $(24a^2 - 43a^3b + 26ab^2 - 29b^3) \div (3a - 5b)$.
10. $(84a^3 - 59a^2b - 53ab^2 + 36b^3) \div (7a - 2b)$.
11. $(6x^4 + 17x^3 + 34x^2 + 33x + 26) \div (2x^2 + 3x + 4)$.
12. $(15x^4 + 31x^3 + 71x^2 + 61x + 59) \div (3x^2 + 2x + 6)$.
13. $(8x^4 - 10x^3 + 31x^2 - 51x + 41) \div (4x^2 - 7x + 3)$.
14. $(35x^4 - 52x^3 + 23x^2 + 41x - 81) \div (5x^2 - 6x + 8)$.
15. $(18x^4 - 39x^3 - 13x^2 + 43x + 7) \div (6x^2 - 5x - 7)$.
16. $(6x^4 - 7x^3y - 21x^2y^2 + 17xy^3 + 15y^4) \div (3x^2 - 2xy - y^2)$.
17. $(12x^4 + 4x^3y - 37x^2y^2 + 85xy^3 - 39y^4)$
$\div (4x^2 + 8xy - 7y^2)$.
18. $(27x^4 + 66x^3 + 85x^2 + 104x + 65) \div (9x + 10)$.
19. $(10x^4 - 31x^3 + 44x^2 - 57x + 56) \div (5x - 8)$.
20. $(-20x^4 - 19x^3 + 16x^2 - 13x + 7) \div (-4x - 7)$.

Find the quotient of;

21. $(a^3 + b^3 + c^3 - 3abc) \div (a + b + c)$.
22. $(a^3 + b^3 - c^3 + 3abc) \div (a + b - c)$.
23. $(a^3 - b^3 + c^3 + 3abc) \div (a - b + c)$.
24. $(-a^3 + b^3 + c^3 + 3abc) \div (-a + b + c)$.
25. $(x^3 + y^3 + 8 - 6xy) \div (x + y + 2)$.
26. $(x^3 + y^3 - 27 + 9xy) \div (x + y - 3)$.
27. $(x^3 - y^3 + 1 + 3xy) \div (x - y + 1)$.
28. $(8x^3 + 27y^3 + 64z^3 - 72xyz) \div (2x + 3y + 4z)$.
29. $(27x^3 - y^3 + 8z^3 + 18xyz) \div (3x - y + 2z)$.
30. $(125x^3 + 27y^3 - 216 + 270xy) \div (5x + 3y - 6)$.

Divide the product of:

31. $x^3 - 12x + 16$ and $x^3 - 12x - 16$ by $x^2 - 16$.
32. $x^3 - 3x + 2$ and $x^2 - 2x + 1$ by $x^3 - 3x^2 + 3x - 1$.
33. $x - 4$, $2x^2 + 3$, $x^2 + x - 1$, and $x^2 - x - 1$ by $x^4 - 3x^2 + 1$.
34. $a^3 + x^3$ and $a^2 + ax + x^2$ by $a^4 + a^2x^2 + x^4$.
35. $x^2 + 2xa + a^2$ and $x^4 - 4x^3a + 6x^2a^2 - 4xa^3 + a^4$ by
$x^4 - 2x^3a + 2xa^3 - a^4$.

E.

Perform the additions and subtractions:

1. $\dfrac{3x - y}{5x + 3y} + \dfrac{x + 3y}{7x + 9y}$.

2. $\dfrac{23x^2 + 18xy + 17y^2}{12x^2 + 31xy + 20y^2} - \dfrac{2x + 3y}{3x + 4y}$.

3. $\dfrac{x - m}{x - n} + \dfrac{x - n}{x - m} - \dfrac{(m - n)^2}{(x - m)(x - n)}$.

4. $\dfrac{x^2 - x + 1}{x^2 + x + 1} + \dfrac{2x(x - 1)^2}{x^4 + x^2 + 1} + \dfrac{2x^2(x^2 - 1)^2}{x^8 + x^4 + 1}$.

5. $\dfrac{1}{1 - a} - \dfrac{1}{1 + a} - \dfrac{2a}{1 + a^2} - \dfrac{4a^3}{1 + a^4} - \dfrac{8a^7}{1 + a^8}$.

6. $\dfrac{a}{2a - 2b} + \dfrac{b}{2b - 2a}$.

7. $\left(\dfrac{1}{m} + \dfrac{1}{n}\right)(x + y) - \left(\dfrac{x + y}{m} - \dfrac{x - y}{n}\right)$.

8. $\dfrac{15}{2(x + 1)} - \dfrac{3}{10(x - 1)} - \dfrac{72}{5(2x + 3)}$.

9. $\dfrac{3 + 2a}{2 - a} - \dfrac{2 - 3a}{2 + a} + \dfrac{16a - a^2}{a^2 - 4}$.

10. $\dfrac{1}{x + y} + \dfrac{y}{x^2 - y^2} - \dfrac{x}{x^2 + y^2}$.

11. $\dfrac{x}{x-a} + \dfrac{3x}{x+a} - \dfrac{2ax}{x^2-a^2}$.

12. $\dfrac{3x-4y}{7} - \dfrac{2x-y-z}{3} + \dfrac{15x-4z}{12} - \dfrac{x-4y}{21}$.

13. $\dfrac{x+8}{x^2+5x+6} + \dfrac{x+7}{x^2+7x+12} + \dfrac{x+6}{x^2+9x+20}$.

14. $\dfrac{x+11}{x^2-8x+15} - \dfrac{x+10}{x^2-11x+24} + \dfrac{x+9}{x^2-13x+40}$.

15. $\dfrac{x-9}{x^2+4x-21} + \dfrac{x-12}{x^2+5x-24} - \dfrac{x-4}{x^2+15x+56}$.

16. $\dfrac{a^3+a^2b}{a^2b-b^3} - \dfrac{a(a-b)}{b(a+b)} - \dfrac{2ab}{a^2-b^2}$.

17. $\dfrac{x+y}{y} - \dfrac{2x}{x+y} + \dfrac{x^2y-x^3}{x^2y-y^3}$.

18. $\dfrac{a}{a^2-b^2} - \dfrac{a}{a^2+b^2} - \dfrac{a^2}{(a+b)(a^2+b^2)} + \dfrac{2a^3+b^3-ab^2}{a^4-b^4}$.

19. $\dfrac{\dfrac{1+2x}{1-2x} - \dfrac{1-2x}{1+2x}}{\dfrac{1-2x}{1+2x} + \dfrac{1+2x}{1-2x}}$.

20. $\dfrac{a^2-(b-c)^2}{(a+c)^2-b^2} + \dfrac{b^2-(c-a)^2}{(a+b)^2-c^2} + \dfrac{c^2-(a-b)^2}{(b+c)^2-a^2}$.

ANSWERS.

B.—(1) $3ab + bc + 4ca$. (2) $39ab - bc - 10ca$. (3) $pq - 3qr + rp$. (4) $6x$. (5) $20a - 2b + 9c$. (6) $39xy - 36yz - zx$. (7) $-2x - 43y + 31z$. (8) $27a - 24bc - 16d$. (9) $9ax - 7by - 4cz$. (10) $19a + 7b - 7c + 13d$.

EXERCISE V.—(Page 9). A.—(1) abc. (2) $13x^2 - 6xy + 7y^2$. (3) $-6a^2 + 6ab + 9b^2$. (4) $10x^2 + 4xy + y^2$. (5) $xy + yz + xz$. (6) $8a^3 - 9a^2 - 6a + 15$. (7) $15x^4 - 8x^3 - 21x^2$. (8) $15x^3 + 16x^2 - 36x - 33$. (9) $a^3 + b^3 + c^3$. (10) $a^3 + b^3 + c^3 + d^3$. (11) $2x^3 + x^2 - 10x + 12$. (12) $3x^2 + 2xy - 3xz - 2y^2 - 4yz$. (13) $2x^3 - 6x^2y + 5xy^2$. (14) $a^3 + b^3 + c^3 - 3abc$. (15) $10x^3 + 38x^2y - 21xy^2 + 22y^3$.

B.—(1) $\frac{1}{4}a - \frac{7}{12}b - \frac{11}{8}c$. (2) $\frac{151}{40}a + \frac{53}{40}b - \frac{107}{48}c$. (3) $\frac{8}{3}x^2 - \frac{29}{12}xy - \frac{13}{40}y^2$. (4) $\frac{1}{12}a + \frac{5}{12}b + \frac{5}{12}c$. (5) $-\frac{4}{3}a + \frac{4}{3}b - \frac{1}{2}c$. (6) $\frac{1}{2}x^2 - 1\frac{1}{5}xy + \frac{1}{10}y^2$. (7) $-\frac{1}{4}x^3 - \frac{5}{8}ax^2 + \frac{5}{8}a^2x$. (8) $-a^3 - \frac{1}{3}a^2b + \frac{1}{4}ab^2 + b^3$. (9) $\frac{155}{48}m + \frac{15}{16}n + \frac{4}{27}p$. (10) $-\frac{2}{3}a^3 - \frac{1}{4}b^3 + \frac{7}{12}c^3 - \frac{62}{15}abc$.

EXERCISE VI.—(Page 11). A.—(1) $5x + 5y + 4z$. (2) $8a + 8b + 2c$. (3) $3a + 10b + 10c$. (4) $2x - 3y - 3z$. (5) $a + b - 5c$. (6) $7x + 17y - 17z$. (7) $-4a + 33b - 3c$. (8) $10x + 13y - 14z$. (9) $6a - 4b + 5c$. (10) $4x + 27y - 30z$. (11) $2ab - 6bc + 7cd$. (12) $-2ab + 2cd - 2ac + 2bd$.

B.—(1) $cd - 8ac + 6bd$. (2) $-2xy + 2yz - 2zx$. (3) $-12p + 19q - r$. (4) $-6a - 2b - 2c$. (5) $2a + 2c - 5$. (6) $-x + 3y + 3z$. (7) $-a - b - 5c$. (8) $4ab + 20xy - 41$. (9) $7a - 13xy + 27$. (10) $2x - \frac{7}{5}y + \frac{8}{7}z$. (11) $-\frac{13}{24}x - 1\frac{3}{4}y - 2\frac{11}{12}$. (12) $-\frac{5}{6}a - \frac{13}{3}b + \frac{1}{2}c$.

EXERCISE VII.—(Page 12). A.—(1) $6xy - 10yz + 21zx$. (2) $-2x^3 + 14x^2 + 12x + 3$. (3) $-9x^2y^2 - 22xy^3 + 25y^4$. (4) $-15 + 7ab - 8a^2b^2$. (5) $-3a^2bc + 6b^2ca - 5c^2ab$. (6) $-2a^2b + 16ab^2 - 37$. (7) $-x^2y + 41$. (8) $-2a^2 - 2d^2$. (9) $9x^3y - 6x^2y^2 - 2xy^3$. (10) $2a^3 - 6a^2b + 6ab^2 - 2b^3$. (11) $34x^3 - 30x^2 + 26x - 22$. (12) $4a^2 + 4ab - 8b^2 + 2c^2$.

B —(1) $a^3 - abc + c^3$. (2) $4x^4 - 3x^3 - 3x^2 - 2x + 3$. (3) $-4a^3 - 6b^3 + 8abc$. (4) $x^5 + 2x^4 + x^3 + x^2 - 2x - 2$. (5) $-a^3 + 9a^2b + 15ab^2$. (6) $-2x^4 - 3x^3 + 2x^2 - 17x + 14$.

(7) $-\frac{1}{2}a + \frac{4}{3}b$.   (8) $\frac{3}{2}x^2 - \frac{29}{27}x - \frac{50}{144}$.   (9) $\frac{3}{4}a^2 - \frac{3}{5}a - 14\frac{1}{4}$.
(10) $1\frac{13}{24}x^2 - x + \frac{2}{21}$.   (11) $\frac{6}{5}x^2 - \frac{1}{2}x^2y + \frac{1}{12}xy^2 - \frac{1}{6}y^2$.   (12)
$-\frac{5}{36}a^2 - 2\frac{1}{4}ax^2 + \frac{11}{10}ax^3$.

EXERCISE VIII.— (Page 13).   (1) $21x - 6y - 7z$.
(2) $4a + b - 31c$.   (3) $-5x^3 - 5x^2 - 3x + 28$.   (4) $3 + 3a$
$-23a^2 - 34a^3$. (5) $11a - 3b + 70c$. (6) $4x^2 - 12xy + 2y^2$.
(7) $4x^3 + x^2y - 13xy^2 + 18y^3$.   (8) $-23a^2 + 27a + 3$.   (9)
$18x^2 - 3x + 5y + 2$.   (10) $-4ab + 5ac - 7bc$.   (11) $-8x^2$
$+9x - 5$.   (12) $-24x^3 - 20x^2 + 38x + 27$.   (13) $3x^2 + 13xy$
$-2y^2 - 16xz - 9yz - 3z^2$.   (14) 0.   (15) $-3x + 14$.
(16) $-3l - 6m - 7n$.

EXERCISE IX.—(Page 14).   A.—(1) $20x^5$.   (2)
$20a^9$. (3) $21x^2y^2$. (4) $4a^2b^2$. (5) $24xy$. (6) $34ab$.
(7) $5a^8$.   (8) $63x^7$.   (9) $30a^3b^3$.   (10) $72x^4y^2$.   (11)
$121a^6b^6$. (12) $32a^7b^3$. (13) $6a^2x^7y^3$. (14) $35a^8b^{13}c^4$.
(15) $abcxyz$.   (16) $72a^2cx^2$.   (17) $21a^4b^4x^3$.   (18)
$30a^3x^6y^2$. (19) $2a^7b^8$. (20) $28x^7y^8z^3$. (21) $48a^5b^3cx$.
(22) $76x^5y^5z^5$.   (23) $98abcxyz$.   (24) $7a^{12}b^7c^6$.   (25)
$abxy^3z^4$. (26) $9m^3n^8p^7$. (27) $56a^5c^8x^3$. (28) ac^2x^2y.
(29) $5a^2b^3c^3x^2$. (30) $224a^{24}b^{15}m^2$.

B.—(1) $a^3b^2 + a^2b^2c$.   (2) $5x^5y^5 + 5x^4y^6$.   (3) $56x^3$
$+21x^2y$. (4) $15a^3b^4x^4 - 27a^2b^4x^5$. (5) $20a^3b - 15ab^2$.
(6) $56x^4 - 63x^3y$.   (7) $-3a^2x^3$.   (8) $27a^2b^2x^2$.   (9)
$3a^3b^3c^4d^5$.   (10) $-8x^3y^4z^5$.   (11) $-a^2b^2c^4$.   (12)
$63x^4y^4z^4$.   (13) $6m^3n^4p^3$.   (14) $9a^2bc$.   (15) a^3bc
$+ab^3c - abc^3$.   (16) $a^3b^4c^4 - a^2b^4c^5 - a^3b^3c^5$.   (17)
$-15a^3c + 27ab^2c + 33c^3$.   (18). $126x^5y + 112x^4y^2$
$-98x^3y^3$. (19) $21x^6y^4 + 35x^4y^6$. (20) $8x^3y^2z^2 - 3x^2y^3z^2$
$+13x^2y^2z^3$.   (21) $a^3b^2c + a^2b^3c + a^2b^2c^2$.   (22) $5x^4y^5z^2$
$-5x^2y^5z^6 + 5x^4y^2z^6$. (23) $8x^2y^4z^4 - 12x^3y^5z^2 - 20x^4y^3z^3$.
(24) $-65x^3 + 52x^2 - 104x + 117$.   (25) $-50a^4b - 70a^3b$
$+60a^2b - 110ab$.

EXERCISE X.—(Page 15). A.—(1) $8x^3 - 26x^2 + 33x$
-18.   (2) $20x^4 + x^3 - 2x^2 + 3x - 40$.   (3) $12a^5 - 37a^4$
$+15a^3 + 11a^2 - 17a + 56$. (4) $6x^4 - 96$. (5) $x^4 - 2x + 1$.
(6) $35 - 47a + 31a^2 - 75a^3 + 54a^4$.   (7) $26 - 115x - 15x^2$
$+13x^3 + 63x^4$.   (8) $15x^3 - 32x^2y + 37xy^2 - 28y^3$.

(9) $49a^4 - 91a^3b + 106a^2b^2 + 2ab^3 - 24b^4$. (10) $x^4 - x^3a + xa^3 - a^4$. (11) $64a^3 - 27b^3$. (12) $40x^4 - 11x^3 - 40x^2 + 83x - 72$. (13) $a^4 + 4a^2x^2 + 16x^4$. (14) $100a^4 - 9a^2x^2 + 6ax^3 - x^4$. (15) $a^5 - 5a^4b + 12a^3b^2 - 16a^2b^3 + 11ab^4 - 3b^5$. (16) $x^6 - 729a^6$. (17) $9x^6 + 14x^4 - 23x^2 - 36$.

B.—(1) $a^2 + 2ab + b^2 - c^2$. (2) $4a^2 + 12ab + 9b^2 - 16c^2$. (3) $x^4 + x^2y^2 + y^4$. (4) $9x^4 - 4x^2y^2 + 16xy^3 - 16y^4$. (5) $x^4 - 2x^2y^2 + y^4$. (6) $a^2b^2 + c^2d^2 - a^2c^2 - b^2d^2$. (7) $a^6 - 1$. (8) $a^6 + 2a^3b^3 + b^6$. (9) $x^3 + y^3 + z^3 - 3xyz$. (10) $x^3 + y^3 - 1 + 3xy$. (11) $81a^4 - 256x^4$. (12) $x^{15} + y^{10}$. (13) $a^6 - 2a^3 + 1$. (14) $x^4 - a^4$. (15) $x^6 + 2x^3y^3 + y^6$. (16) $x^8 + x^4a^4 + a^8$.

EXERCISE XI.—(Page 16). A.—(1) x^3. (2) x^7. (3) $3a^2$. (4) $9a$. (5) $5x^3$. (6) $-9a^3$. (7) $-8x^3$. (8) xy^2. (9) a^5x^3. (10) $4ac^2$. (11) $-5a^2b^4c^5$. (12) $5x^4y$. (13) $6a^7$. (14) $7a^5$. (15) $7a^2b^2c^2$. (16) -1. (17) $10y^2x$. (18) $-6b^2x$. (19) $8abc^6$. (20) $-77a^3b^3x^3$.

B.—(1) $2x^4 + 3x^3 + 4x + 5$. (2) $-a^2 + 5a + 6$. (3) $-5a^2b^2 + 2ab - 4$. (4) $9a^2b^2 - 12ab^2c + 15a^2bc$. (5) $-a + b - c$. (6) $a^3 - a^2b + ab^2 - b^3$. (7) $-2x^2y^2 + 3xy - 4$. (8) $8x^4y^4 - 5x^3y - 2x$. (9) $27m^6n^5 - 28m^3n^4 + 9mn^2p$. (10) $13a^2b - 9ab^2 - 7b$. (11) $19b^2c^2 + 12bc^3 - 7c^3$. (12) $12x^3 - 9x^2y + 8xy^2 - 5y^3$.

EXERCISE XII.—(Page 17). A.—(1) $x + 6$. (2) $x - 10$. (3) $a - 5$. (4) $x - 24$. (5) $3a + 1$. (6) $5a + 1$. (7) $x + 5$. (8) $4a - 7$. (9) $3a - 5$. (10) $5x - a$. (11) $3a + 7c$. (12) $10x - 9y$. (13) $8x + 3y$. (14) $3a^2 + 2a + 1$. (15) $x^2 - 3x + 7$. (16) $x^2 + x + 1$. (17) $x^2 + 3y$. (18) $a^3 + 3a^2b + 3ab^2 + b^3$. (19) $a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$. (20) $x^2 - 2x + 1$.

B.—(1) $x^2 - 2x + 2$. (2) $a^2 - 3a - 1$. (3) $x^2 + 5x + 6$. (4) $6a^2 - 7a + 8$. (5) $7a^2 + 5ab + 2b^2$. (6) $x^2 - 2x + 3$. (7) $x^2 + 8x + 12$. (8) $m^3 + 7m - 5$. (9) $a^2 + 2ab + b^2 + a + b + 1$. (10) $b^5 + 5b^4c + 10b^3c^2 + 10b^2c^3 + 5bc^4 + c^5$. (11) $3a + 2b + c$. (12) $x^4 - x^3y + x^2y^2 - xy^3 + y^4$. (13) $a^4 + a^3b + a^2b^2 + ab^3 + b^4$. (14) $x^5 + x^4y + x^3y^2 + x^2y^3$

$+xy^4+y^5$. (15) $a^3-2a^2b+4ab^2-8b^3$. (16) $27x^3$
$-18x^2y+12xy^2-8y^3$. (17) $8a^3+12a^2b+18ab^2+27b^3$.
(18) $x^3+3x^2y+9xy^2+27y^3$. (19) $x^6-x^5+x^4-x^3+x^2$
$-x+1$. (20) $x^3+2x^2y+2xy^2+y^3$. (21) $a^8-a^6+2a^2-2$.
(22) $x^2+2xy+y^2-xz-yz+z^2$. (23) $a+b+c$. (24) $a+b$
$-c-d$. (25) $x^2+y^2+z^2-xy-xz-yz$.

EXERCISE XIII.—(Page 18). A.—(1) a. (2)
$a+b-c$. (3) $a-b$. (4) $2x$. (5) $a+a^3$. (6) $-2b+2c$.
(7) $3a-b-c$. (8) $a+3b-4c$. (9) $5a$. (10) $4a$. (11) x.
(12) $-x-2y+6z$. (13) $-5a$. (14) $2a+4b$. (15) $11x$
$-36y$. (16) $21a+b$. (17) $2x-3y+12z$. (18) $-a^2$
$+8b^2-9c^2$. (19) $-50c$. (20) $-a-10b+2c$. (21) $x+c$.
(22) x^2-ax+b. (23) $x^2-(a-2b)x-2ab$. (24) x^2-px+q.
(25) $x^2+ax-2b$. (26) x^2+bx+a^2. (27) x^3-ax^2+bx.
(28) $x^4+(p-q)x^2+pq$. (29) px^2+qx+r. (30) x^2
$+(n+1)ax-a^2$.

B.—(1) $(ax-bx+cx)-(ay-by+cy)$; $(ax-ay)$
$-(bx-by)+(cx-cy)$. (2) $(ax^3-dx^3)+(bx^2-dx^2)$
$+(bx-cx-2x)+(7-c)$. (3) $-(a^2x-a^2y)-(7a+ab)$
$-(2x-3)$. (4) $(ax^4+3x^4)+(bx^2-8x^2)+(3bx-9x)+7$.
(5) $(6ax^3-bx^3)+(4bx^2-2x^2)+(cx-5x)+(ab-8)$. (6)
$(10ax^3-8x^3)+(6ax^2-12x^2)+(9x-3cx)+4$. (7) $(3cx^5$
$-2a^2x^5)+(3x^4-4bx^4)+5dx-4abc$. (8) $-(bx^4+2a^2x^4)$
$-(3bx^3-4x^3)-(3x^2-ax^2)$. (9) $-(abx^5-7x^5)-(abcx^3$
$-8x^3)-(3c^2x-9ax)$. (10) $-(cx^3-a^2x^3)-(bx^2-ax^2$
$+5x^2)$. (11) $-(3ax^4-6b^2x^4+cx^4+7x^4)-(2bx+5c^2x)$.
(12) $-(-5ax^3-4cx^3)-(-3ax^2+6bx^2-7cx^2)-(-2ax$
$+7bx)$. (13) $(2a-3b)-(4c-5d)-(4e-3f)$; $(2a-3b$
$-4c)+(5d-4e+3f)$. (14) $-(b+5c)+(6d-3e)+(4f$
$+g)$; $-(b+5c-6d)-(3e-4f-g)$. (15) $-(3x-4y)$
$-(2z-3a)+(2b-c)$; $-(3x-4y+2z)+(3a+2b-c)$.
(16) $(4c-2d)+(3e+2x)-(y+5z)$; $(4c-2d+3c)+(2x-y$
$-5z)$. (17) $-(2m-3n)+(4a-6b)-(5x-7y)$; $-(2m$
$-3n-4a)-(6b+5x-7y)$. (18) $(3p+2q)-(4r+5m)$
$+(3n-2a)$; $(3p+2q-4r)-(5m-3n+2a)$.

EXERCISE XIV.—(Page 20). A.—(1) 3. (2) 2.
(3) 9. (4) 7. (5) 11. (6) 7. (7) 2. (8) 2. (9) 3.
(10) 4. (11) 9. (12) $56\frac{1}{2}$. (13) $2\frac{2}{3}$. (14) 9. (15) $-4\frac{1}{4}$.

(16) -3. (17) 2. (18) 7. (19) 4. (20) 75. (21) 8.
(22) 7. (23) 10. (24) $\frac{5}{6}$. (25) 4.

B.—(1) 13. (2) 5. (3) 16. (4) 10. (5) 4. (6) 15.
(7) $-$1. (8) 1. (9) 2. (10) 1. (11) 1. (12) 1.
(13) 2. (14) 1. (15) 20. (16) 3. (17) 12. (18) 19.
(19) 1. (20) 2. (21) 2. (22) 4. (23) 7. (24) $-6\frac{1}{3}$.
(25) $5\frac{1}{6}$.

EXERCISE XV.—(Page 21). A.—(7) $x-(a+b)$.

(8) $x+y+z$. (9) $na+x$. (10) $\dfrac{m-(ax+by)}{z}$. (11) $\dfrac{x}{a+b}$

hours; $\dfrac{ax}{a+b}$ and $\dfrac{bx}{a+b}$ miles.

B.—(1) abc; $2ab+2ac+2bc$; $4a+4b+4c$. (2) $432xy$.

(3) $\$ \{ 2(a+b)c+ab \} \dfrac{x}{900}$. (4) $(mp+nq)\dfrac{x}{100}$. (5)

$\dfrac{ax+by+cz}{a+b+c}$. (6) $(x+y)-\{ n-(x+y) \}$.

EXERCISE XVI. —(Page 24). A.—(1) 22 ; 16.
(2) 55 ; 28. (3) 54 ; 41. (4) 158 ; 131. (5) 39.
(6) 36 ; 24. (7) 519 ; 456. (8) 79 ; 53. (9) 176 ; 115.
(10) 379. (11) 2. (12) $94. (13) 27 ; 36. (14) 84 ;
85 ; 86. (15) $765 ; $819 ; $842. (16) 53 ; 17. (17) 8.
(18) 103. (19) $350 ; 450 ; 720. (20) 74.

B.—(1) $260. (2) 36 ; 24. (3) $200 ; 500. (4) 12.
(5) 15 ; 12. (6) 10 ; 18. (7) 39. (8) $90. (9) 15 ; 21 ;
6 ; 54. (10) $60. (11) 6 ; 3. (12) 31. (13) 51.
(14) 50; 51 ; 52. (15) £49.

EXERCISE XVII.—(Page 27). A.—(1) $3x^2$. (2) $7a^4$.
(3) a^2b. (4) xyz. (5) $2x^2y$. (6) $15m^2n^3$. (7) $4abc$.
(8) $17p^2qr$. (9) abc. (10) x^2z^2. (11) $7a^2b^4c^2$.
(12) $7a^2b^2x^3y^3$. (13) $7a$. (14) $17abc$. (15) $2ax$.
(16) xy. (17) $8a^2b^2c^2$. (18) $25xy$. (19) xy. (20)
$5a^3b^4c^2$. (21) a^2bc. (22) $11p^2m$. (23) $12x^3y^3z^4$.
(24) y.

EXERCISE XVIII.—(Page 28). (1) $\dfrac{2}{3a}$. (2) $\dfrac{2x}{9}$.

(3) $\dfrac{5b}{12a}$. (4) $\dfrac{4xy}{3bc}$. (5) $\dfrac{2x^2y}{5z^2}$. (6) $\dfrac{bc^6}{3a^4}$. (7) $\dfrac{3}{2az}$.

(8) $\dfrac{3}{4x^2y^6z}$. (9) $\dfrac{6ac}{7b}$. (10) $\dfrac{5m}{2np}$. (11) $\dfrac{2a^2x}{3b^2y}$.

(12) $\dfrac{4ac}{5bd}$. (13) $\dfrac{3cz}{4axy}$. (14) $\dfrac{3a}{4b}$. (15) $\dfrac{2x}{5y}$. (16) $\dfrac{4a^2}{5bc}$.

(17) $\dfrac{4n}{5mp}$. (18) $\dfrac{c}{a^2b}$. (19) $\dfrac{3xz}{5y}$. (20) $\dfrac{q}{mnp}$. (21) $\dfrac{3b}{4ac}$.

(22) $\dfrac{2m}{3kp}$. (23) $\dfrac{2xyz}{3}$. (24) $\dfrac{ab}{xycd}$.

EXERCISE XIX.—(Page 29). (1) $\dfrac{7}{12}$. (2) $\dfrac{1}{4}$.

(3) $\dfrac{2x}{3y}$. (4) $\dfrac{2by}{3ax}$. (5) ax. (6) $\dfrac{3cd^2}{2b}$. (7) $\dfrac{9a^2}{8b^2c^2}$.

(8) $\dfrac{ax^2z^2}{9bc}$. (9) $\dfrac{14ab^2}{15cy}$. (10) $\dfrac{3nz}{2m}$. (11) $\dfrac{5mp^2}{2n}$.

(12) $\dfrac{6x^2z^2}{bc}$. (13) 1. (14) $\dfrac{a^3b^3c^3}{x^3y^3z^3}$. (15) $\dfrac{8a^2c^2}{9d^2}$.

(16) $\dfrac{125km^2}{12pq}$. (17) $\dfrac{y^3z^2}{nx^4}$. (18) $\dfrac{7b}{4a}$. (19) $\dfrac{d}{4a^2c}$.

(20) $\dfrac{49a^2cy^2}{64bd^2x^2}$. (21) $\dfrac{2np^4q^5y^3}{25m^2x^4}$. (22) $\dfrac{ax^2y}{b^5p}$. (23) $\dfrac{9c}{8x}$.

(24) $\dfrac{108x^4y^2z^2}{125a^2}$.

EXERCISE XX.—(Page 30). (1) $12a^2b^2$. (2) $36a^3b^2c^3$. (3) $24a^2b^2x^3y^3$. (4) $12a^3x^2$. (5) $12x^5y^3$. (6) $8a^2b^3c^4$. (7) $2a^2bc$. (8) $24abxy$. (9) $15a^4b^4c^5$. (10) $12x^3y^3z$. (11) $228a^3b^2x^3y$. (12) $81m^3n^2pq$. (13) $24a^2bc$. (14) $a^2b^2c^2$. (15) $12x^3y^4$. (16) $42x^4y^5$. (17) $210a^3b^3c^3$. (18) $120a^2b^2c^2$. (19) $132a^4b^4c^3$.

(20) $2520abcxyz$.   (21) $60mnpq$.   (22) $780a^2b^2c^2$.
(23) $204xyz$.   (24) $1140abx^2y^3z^2$.

EXERCISE XXI. – (Page 30).   (1) $\dfrac{7x}{12}$.   (2) $\dfrac{5a}{6}$.

(3) $\dfrac{7x+5y}{35}$.   (4) $\dfrac{34x}{35}$.   (5) $\dfrac{3a+b}{39}$.   (6) $\dfrac{a}{126}$.

(7) $\dfrac{9x}{20}$.   (8) $\dfrac{5m}{18}$.   (9) $\dfrac{y}{24}$.   (10) $\dfrac{7ab}{24}$.   (11) $\dfrac{11x}{30}$.

(12) $\dfrac{a}{6}$.   (13) $\dfrac{7x}{18}$.   (14) $\dfrac{3x}{4}$.   (15) $\dfrac{59x}{36}$.   (16) $\dfrac{23x}{48}$.

(17) $\dfrac{19x}{54}$.   (18) $\dfrac{15a+8b+14c}{24}$.   (19) $\dfrac{5a+b}{42}$.

(20) $\dfrac{5x-y}{54}$.   (21) $\dfrac{ab+y}{b}$.   (22) $\dfrac{9bx+8ay}{6ab}$.

(23) $\dfrac{14ay-5bx}{24xy}$.   (24) $\dfrac{acx-by}{cx}$.   (25) $\dfrac{2ax^2-b^2}{a}$.

(26) $\dfrac{2ac^2x-3y}{a^3c^4}$.   (27) $\dfrac{9ac+5b^2}{21bc}$.   (28) $\dfrac{2x-12cy}{3y}$.

(29) $\dfrac{2m+3n-6p}{6x}$.   (30) $\dfrac{15a-12b-10c}{60n}$.

EXERCISE XXVI.—(Page 34).   (1) c^2; $b+c$.
(2) 9; $c+3$.   (3) q^2; $p+q$.   (4) $25q^2$; $p+5q$.
(5) 1; $x+1$.   (6) 9; $ab+3$.   (7) 16; $y-4$.   (8) 36; $x-6$.   (9) $49b^2$; $a-7b$.   (10) $100z^2$; $x-10z$.   (11) $9b^2$; $2a+3b$.   (12) $16q^2$; $3p+4q$.   (13) 9; $4x+3$.
(14) 361; $y+19$.   (15) 1; $6a+1$.   (16) $4b^2$; $5a+2b$.
(17) c^2; $2a+c$.   (18) y^2; $9x-y$.   (19) $4b^2$; $3a-2b$.
(20) $9d^2$; $2c-3d$.   (21) $9y^2$; $5x-3y$.   (22) a^2; $1-a$.
(23) $4x^2$; $a-2x$.   (24) $49c^2$; $2ax-7c$.   (25) $25x^2y^2$; $3am+5xy$.   (26) 1; $4c-1$.   (27) 9; $2xy+3$.   (28) 1; $ax-1$.   (29) 4; $by+2$.   (30) 4; $13xz-2$.

EXERCISE XXVII.—(Page 34).   (21) $x^2+2xy+y^2-z^2$.   (22) $a^2+2ab+b^2-c^2$.   (23) $l^2+2lm+m^2$

$-n^2$. (24) $4a^2 + 12ab + 9b^2 - 16c^2$. (25) $25x^2 - 30xz$ $- 4y^2 + 9z^2$. (26) $9a^2 + 6ac - 16b^2 + c^2$. (27) $x^4 + x^2 + 1$. (28) $a^4x^4 + a^2x^2 + 1$. (29) $x^4 + x^2y^2 + y^4$. (30) $a^4 + a^2b^2 + b^4$.

EXERCISE XXVIII.—(Page 35). C.—(1) $a+b+c)$ $(a+b-c)$. (2) $(x-y+z)(x-y-z)$. (3) $(a-b+x)$ $(a-b-x)$. (4) $(a+b-c)(a-b+c)$. (5) $(1+m+n)$ $(1-m-n)$. (6) $(x+y+3xy)(x+y-3xy)$. (7) $a+b$ $+2ab)(a+b-2ab)$. (8) $(x+a+y)(x+a-y)$. (9) $(y+c$ $-x)(y-c+x)$. (10) $(x-2y+3xy)(x-2y-3xy)$. (11) $(a-5b+1)(a-5b-1)$. (12) $(4x+a+3b)(4x-a-3b$. (13) $(2m+p-q)(2m-p+q)$. (14) $(p+2q+r)(p+2q-r)$. (15) $(x+y+z)(x-y+z)$. (16) $(a+b+c-d)(a+b-c$ $+d)$. (17) $(a-b+c+d)(a-b-c-d)$. (18) $(2a+6b$ $+4c)(2a-4c)$. (19) $(x-y+b-2c)(x-y-b+2c)$. (20) $(a+b+3x+y)(a-b+3x-y)$. (21) $(x+m+2n-1)$ $(x-m-2n-1)$. (22) $(a+b+c+d)(a+b-c-d)$. (23) $(a+b-c-d)(a-b-c+d)$. (24) $(a+x+y+z)(a+x-y$ $-z)$. (25) $(a-4b+5c-1)(a-4b-5c+1)$. (26) $(3ab$ $-4cd+x+y)(3ab+4cd+x-y)$. (27) $(x-y+z+w)$ $(x-y-z-w)$. (28) $2ab+3cd+4xy)(2ab+3cd-4xy$. (29) $(5ab+6c-9d)(5ab-6c+9d)$. (30) $(2x-3a+c+d)$ $(2x-3a-c-d)$.

EXERCISE XXIX.—(Page 36). (1) 350000. (2) 136100. (3) 300000. (4) 2452000. (5) 336000. (6) 21320000. (7) 25930800. (8) 4000. (9) 81735. (10) 236880.

EXERCISE XXX.—(Page 36). (1) $a^2 + b^2 + c^2$ $+2ab+2ac+2bc$. (2) $x^2 + y^2 + z^2 + 2xy + 2xz + 2yz$. (3) $m^2 + n^2 + p^2 + 2mn + 2np + 2np$. (4) $a^2 + b^2 + c^2$ $+2ab - 2ac - 2bc$. (5) $x^2 + y^2 + z^2 - 2xy + 2xz - 2yz$. (6) $a^2 + b^2 + c^2 - 2ab - 2ac + 2bc$. (7) $x^2 + 4y^2 + z^2 + 4xy$ $+2xz + 4yz$. (8) $9a^2 + 16b^2 + 4c^2 - 24ab + 12ac - 16bc$. (9) $1 + 4x^2 + 9y^2 + 4x - 6y - 12xy$. (10) $25 + 64y^2 + 36z^2$ $- 80y + 60z - 96yz$. (11) $16 - 40x - 23x^2 + 60x^3 + 36x^4$. (12) $9x^4 + 42x^3 + x^2 - 112x + 64$. (13) $25p^2 + 16q^2 + 81r^2$ $- 40pq + 90pr - 72qr$. (14) $x^4 + y^4 + z^4 + 2x^2y^2 + 2x^2z^2$ $+2y^2z^2$. (15) $a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 + 2b^2c^2$.

(16) $x^2y^2 + y^2z^2 + z^2x^2 + 2xy^2z + 2x^2yz + 2xyz^2$.   (17)
$25x^4 - 90x^3 + 111x^2 - 54x + 9$.   (18) $1 + a^2x^2 + b^2y^2 - 2ax$
$- 2by + 2abxy$ (19) $4(x^2 + y^2 + z^2)$. (20). $4(a^2 + b^2 + c^2)$.
(21) $2x^2y^2 + 2x^2z^2 + 2y^2z^2 - x^4 - y^4 - z^4$.   (22) $2a^2b^2$
$+ 2a^2c^2 + 2b^2c^2 - a^4 - b^4 - c^4$.   (23) $a^2 + b^2 + c^2 - ab$
$- ac - bc$.   (24) $6a^2 + 2b^2 + 2c^2 + 4ab + 4ac$.   (25) $5x$
$- 6y - 7z$.

EXERCISE XXXI.—(Page 37). A.—(1) $x^3 + 3x^2y$
$+ 3xy^2 + y^3$.   (2) $a^3 + 3a^2b + 3ab^2 + b^3$.   (3) $x^3 + 3x^2a$
$+ 3xa^2 + a^3$.   (4) $x^3 - 3x^2a + 3xa^2 - a^3$.   (5) $x^3 + 3x^2z$
$+ 3xz^2 + z^3$. (6) $x^3 - 9x^2 + 27x - 27$. (7) $a^3 + 12a^2 + 48a$
$+ 64$.   (8) $a^3 - 15a^2 + 75a - 125$. (9) $x^3 + 6x^2y + 12xy^2$
$+ 8y^3$.   (10) $a^3 - 6a^2b + 12ab^2 - 8b^3$.   (11) $27x^3 + 27x^2y$
$+ 9xy^2 + y^3$.   (12) $64x^3 - 240x^2y + 300xy^2 - 125y^3$.
(13) $a^3b^3 + 3a^2b^2c + 3abc^2 + c^3$.   (14) $8x^3y^3 - 36x^2y^2z$
$+ 54xyz^2 - 27z^3$.   (15) $216a^3 + 108a^2bc + 18ab^2c^2 + b^3c^3$.
(16) $125x^6 - 300x^4y^2 + 240x^2y^4 - 64y^6$.   (17) $27a^6$
$- 54a^4b^2 + 36a^2b^4 - 8b^6$.   (18) $125x^9 + 75x^6 + 15x^3 + 1$.
(19) $343a^6 - 147a^4 + 21a^2 - 1$.   (20) $a^3x^3 - 3a^2x^2y^2$
$+ 3axy^4 - y^6$.

B.—(1) $a^3 + b^3 + c^3 + 3a^2b + 3ab^2 + 3a^2c + 3ac^2 + 3b^2c$
$+ 3bc^2 + 6abc$.   (2) $x^3 + y^3 + z^3 + 3x^2y + 3xy^2 + 3x^2z$
$+ 3xz^2 + 3y^2z + 3yz^2 + 6xyz$.   (3) $a^3 + b^3 - c^3 + 3a^2b$
$+ 3ab^2 - 3a^2c + 3ac^2 - 3b^2c + 3bc^2 - 6abc$.   (4) $x^3 + y^3$
$- z^3 + 3x^2y + 3xy^2 - 3x^2z + 3xz^2 - 3y^2z + 3yz^2 - 6xyz$.   (5)
$x^3 - y^3 + z^3 - 3x^2y + 3xy^2 + 3x^2z + 3xz^2 + 3y^2z - 3yz^2$
$- 6xyz$.   (6) $a^3 - b^3 + c^3 - 3a^2b + 3ab^2 + 3a^2c + 3ac^2$
$+ 3b^2c - 3bc^2 - 6abc$.   (7) $8x^3 + 27y^3 + 64z^3 + 36x^2y$
$+ 54xy^2 + 48x^2z + 96xz^2 + 108y^2z + 144yz^2 + 144xyz$. (8)
$64a^3 - 27b^3 + 8c^3 - 144a^2b + 108ab^2 + 96a^2c + 48ac^2$
$+ 54b^2c - 36bc^2 - 144abc$.   (9) $125x^3 - 64y^3 - 343z^3$
$- 300x^2y + 240xy^2 - 525x^2z + 735xz^2 - 336y^2z - 588yz^2$
$+ 840xyz$.   (10) $216a^3 - 125b^3 - 64c^3 - 540a^2b + 450ab^2$
$- 432a^2c + 288ac^2 - 300b^2c - 240bc^2 + 720abc$. (11) $27p^3$
$+ 64q^3 - 512r^3 + 108p^2q + 144pq^2 - 216p^2r + 576pr^2$
$- 384q^2r + 768qr^2 - 576pqr$.   (12) $27a^3 - 8b^3 + 125x^3$
$- 54a^2b + 36ab^2 + 135a^2x + 225ax^2 + 60b^2x - 150bx^2$
$- 180abx$.   (13) $x^6 + y^6 + z^6 + 3x^4y^2 + 3x^2y^4 + 3x^4z^2$

$+ 3x^2z^4 + 3y^4z^2 + 3y^2z^4 + 6x^2y^2z^2$. (14) $a^9 - b^9 - c^9$
$- 3a^6b^3 + 3a^3b^6 - 3a^6c^3 + 3a^3c^6 - 3b^6c^3 - 3b^3c^6$
$+ 6a^3b^3c^3$. (15) $8x^6 - 125y^6 + 16z^6 - 60x^4y^2 + 150x^2y^4$
$+ 48x^4z^2 + 96x^2z^4 + 300y^4z^2 - 240y^2z^4 - 240x^2y^2z^2$. (16)
$a^9x^3 - b^9y^3 + c^9z^3 - 3a^2x^2by + 3axb^2y^2 + 3a^2x^2cz$
$+ 3axc^2z^2 + 3b^2y^2cz - 3byc^2z^2 - 6axbycz$. (17) $a^3 + b^3$
$+ c^3 + d^3 + 3a^2b + 3ab^2 + 3a^2c + 3ac^2 + 3a^2d + 3ad^2 + 3b^2c$
$+ 3bc^2 + 3b^2d + 3bd^2 + 3c^2d + 3cd^2 + 6abc + 6abd + 6acd$
$+ 6bcd$. (18) Work by substitution.

EXERCISE XXXII.—(Page 37). (1) $x^3 + (a + b + c)x^2 + (ab + ac + bc)x + abc$. (2) $x^3 + (a - b + c)x^2 + (-ab + ac - bc)x - abc$. (3) $x^3 + (a - b - c)x^2 + (-ac + bc)x + abc$. (4) $x^3 + 6x^2 + 11x + 6$. (5) $x^3 + 15x^2 + 74x + 120$. (6) $a^3 - 12a^2 + 41a - 42$. (7) $x^3 + 14x^2 + 55x + 42$. (8) $x^3 - 6x^2 - 67x + 360$. (9) $8x^3 + 60x^2 + 142x + 105$. (10) $8x^3 - 72x^2 + 208x - 192$. (11) $27x^3 + 117x^2 + 141x + 35$. (12) $27x^3 - 63x^2 - 114x + 240$. (13) $64a^3 - 64a^2 - 156a - 54$. (14) $x^4 + 14x^3 + 7 \cdot x^2 + 154x + 120$. (15) $a^4 + 20a^3 + 137a^2 + 358a + 240$. (16) $x^4 - 2x^3 - 113x^2 + 114x + 3024$.

EXERCISE XXXIII.—(Page 38). A.—(1) $x^2 - xy + y^2$. (2) $a^2 - ab + b^2$. (3) $x^2 - xa + a^2$. (4) $b^2 - by + y^2$. (5) $x^2 - x + 1$. (6) $a^2 - a + 1$. (7) $1 - c + c^2$. (8) $1 - z + z^2$. (9) $x^2 - 2x + 4$. (10) $a^2 - 3a + 9$. (11) $x^2 - 2xy + 4y^2$. (12) $9a^2 - 3ab + b^2$. (13) $4x^2 - 6xb + 9b^2$. (14) $x^2 + xy + y^2$. (15) $a^2 + ab + b^2$. (16) $a^2 + 5a + 25$. (17) $36 + 6x + x^2$. (18) $16x^2 + 20xy + 25y^2$. (19) $m^4 - m^3n + m^2n^2 - mn^3 + n^4$. (20) $a^4 - a^3b + a^2b^2 - ab^3 + b^4$. (21) $x^6 - x^5y + x^4y^2 - x^3y^3 + x^2y^4 - xy^5 + y^6$. (22) $a^6 + a^5b + a^4b^2 + a^3b^3 + a^2b^4 + ab^5 + b^6$. (23) $m^6 + m^5n + m^4n^2 + m^3n^3 + m^2n^4 + mn^5 + n^6$. (24) $x^4 - x^3 + x^2 - x + 1$. (25) $c^4 + c^3 + c^2 + c + 1$. (26) $a^4 - 2a^3 + 4a^2 - 8a + 16$. (27) $x^4 - 3x^3 + 9x^2 - 27x + 81$. (28) $x^4 - x^2a + a^2$. (29) $a^2 + 2ab + b^2 - ac - bc + c^2$. (30) $x^2 - 2xy + y^2 + xz - yz + z^2$.

B.—(1) $x^3 + x^2y + xy^2 + y^3$. (2) $a^5 + a^4b + a^3b^2 + a^2b^3 + ab^4 + b^5$. (3) $m^7 + m^6n + m^5n^2 + m^4n^3 + m^3n^4 + m^2n^5 + mn^6 + n^7$. (4) $x^5 + x^4 + x^3 + x^2 + x + 1$. (5)

$1 + a + a^2 + \ldots\ldots + a^7$. (6) $8x^3 - 12x^2y + 18xy^2 - 27y^3$.
(7) $125a^3 - 100a^2b + 80ab^2 - 64b^3$. (8) $x^5 - x^4y + x^3y^2$
$- x^2y^3 + xy^4 - y^5$. (9) $a^3 - 2a^2b + 4ab^2 - 8b^3$. (10)
$(a+b)^3 - (a+b)^2 + (a+b) - 1$.

EXERCISE XXXIV. — (Page 39). (1) $(x + y)$
$(x^2 - xy + y^2)$. (2) $(x-y)(x^2+xy+y^2)$. (3) $(a-b)$
$(a^2 + ab + b^2)$. (4) $(a + 1)(a^2 - a + 1)$. (5) $(1 + a)$
$(1 - a + a^2)$. (6) $(2x - y)(4x^2 + 2xy + y^2)$. (7) $(a + 2b)$
$(a^2 - 2ab + 4b^2)$. (8) $(3x+2)(9x^2 - 6x + 4)$. (9) $(a-c)$
$(a^3 + a^2c + ac^2 + c^3)$. (10) $(a - 1)(a^7 + a^6 + \ldots\ldots + 1)$.
(11) $(p - q)(p^5 + p^4q + p^3q^2 + p^2q^3 + pq^4 + q^5)$. (12)
$(x - 3)(x^5 + 3x^4 + 9x^3 + 27x^2 + 81x + 243)$. (13) $(ab + c)$
$(a^2b^2 - abc + c^2)$. (14) $(3x - 2y)(9x^2 + 6xy + 4y^2)$. (15)
$(c - 6)(c^2 + 6c + 36)$. (16) $(5 + z)(25 - 5z + z^2)$ (17)
$(xy + 8)(x^2y^2 - 8xy + 64)$. (18) $(7a - 1)(49a^2 + 7a + 1)$.
(19) $(5x - 10y)(25x^2 + 50xy + 100y^2)$. (20) $(x - y)$
$(x^6 + x^5y + \ldots\ldots + y^6)$. (21) $(a + b)(a^4 - a^3b + a^2b^2$
$- ab^3 + b^4)$. (22) $(n+1)(n^6 - n^5 + \ldots\ldots + 1)$. (23)
$(1 - c)(1 + c + c^2 + c^3)$. (24) $(pq + 3r)(p^2q^2 - 3pqr$
$+ 9r^2)$.

EXERCISE XXXV.—(Page 39). A.—(1) $5x(x - 3)$.
(2) $a^2(a - 1)$. (3) $3x^2(1 - 2x)$. (4) $4x(2 - x)$. (5)
$7ax(a - 2x)$. (6) $p(6p + 1)$. (7) $x(x + y)$. (8) $x^4(x - y^2)$.
(9) $3xy^2(x - 3y^2)$. (10) $8(2 + 3x^2)$. (11) $4(3ab - 2)$.
(12) $17(2 + 3x^2y)$. (13) $15a^2(a^2 - 15)$. (14) $5x^3(5y - 2)$.
(15) $3a(8a - 9b^2)$. (16) $3ab(4a^2 - 3b)$. (17) $3x^2y$
$(5x^2 - 2y^2)$. (18) $4m^2n^2p(2m + 5n)$. (19) $x(x^2 + xy - y^2)$.
(20) $2x^2y^2(2y - 3 + 2x)$. (21) $5x^3(3x^2 - 2a^2 - a^3x)$.
(22) $19a^2x^3(2x^2 + 4ax + 3a^2)$. (23) $3ab(a^2 - ab + b^2)$.
(24) $5a(4x^2 - 8x + 9)$. (25) $x^3(6 - 9x + 4x^3)$. (26) $35x^2$
$(m^2 - 2m + 3x)$. (27) $7a(1 - 2a + a^3)$. (28) $11(2m^3 - 3mn$
$+ 4n^2)$. (29) $13p(3pq^2 + 2q - 4p^2)$. (30) $ax(x^3 - bx + c)$.
(31) $7a^2b^2c^2(2bc^2 + ab^2 - 3a^2c)$.

B.—(1) $(a+b)(a+c)$. (2) $(a+b)(a-c)$. (3) $(x+y)$
$(x+z)$. (4) $(x-y)(x-z)$. (5) $(ab+c)(ab+d)$. (6)
$(ac-b)(ac-d)$. (7) $(a+b)(a+4)$. (8) $(x+y)(m-n)$.
(9) $(2x+3y)(a-b)$. (10) $(p-q)(r+s)$. (11) (x^2+1)
$(x+1)$. (12) $(x+a)(m-n)$. (13) $(3a-2b)(5x-4y)$.

(14) $(2x-y)(x-5)$.　　(15) $(3x+2y)(ax+by)$.　　(16)
$(xy-z)(a-bc)$.　(17) $(x^2+y^2)(a^2+b^2)$.　(18) (y^2+1)
$(y-1)$.　(19) $(x^3+3)(x+1)$.　(20) $(a+b+c)(x-y)$.

EXERCISE XXXVI.—(Page 40).　A.—(1) $(x+2)$
$(x+1)$.　(2) $(a+3)(+4)$.　(3) $(x+12)(x+8)$.　(4) $(a+6)$
$(a+17)$.　(5) $(p+12)(p+18)$.　(6) $(xy+5)(xy+26)$.　(7)
$(11+x)(7+x)$.　(8) $(x+6)(x+7)$.　(9) $(x+2y)(x+3y)$.
(10) $(a+2b)(a+7b)$.　(11) $(x+24y)(x+25y)$.　(12) $(m+13n)$
$(m+30n)$.　(13) $(p+8q)(p+27q)$.　(14) $a(1+2a)$
$(1+3a)$.　(15) $3(x+7)(x+8)$.　(16) $(x-5)(x-6)$.　(17)
$(x-9)(x-10)$.　(18) $(x-7)(x-8)$.　(19) $(a-10)(a-11)$.
(20) $(a-3)(a-15)$.　(21) $(a-8)(a-13)$.　(22) $(m-7n)$
$(m-15n)$.　(23) $(xy-4)(xy-13)$.　(24) $(xy-11)(xy-12)$.
(25) $(a-7b)(a-13b)$.　(26) $(a-6bc)(a-15bc)$.　(27)
$(11-x)(10-x)$.　(28) $(ax-11)(ax-12)$.　(29) $7(x-7)$
$(x-8)$.　(30) $(ax-7)(ax-14)$.

B.—(1) $(x+16)(x-5)$.　(2) $(x+11)(x-10)$.　(3)
$(a+26)(a-10)$.　(4) $(ax+24)(ax-10)$.　(5) $(a+12y)$
$(x-5y)$.　(6) $(a+7b)(a-6b)$.　(7) $(m+13)(m-12)$.
(8) $(a+15bx)(a-3bx)$.　(9) $(x^2+18a^2)(x^2-12a^2)$.
(10) $3a(x+17y)(x-5y)$.　(11) $(x-15)(x+6)$.　(12)
$(x-19)(x+8)$.　(13) $(a-35b)(a+3b)$.　(14) $(a-24b)$
$(a+4b)$.　(15) $(m-13)(m+2)$.　(16) $(m-8n)(m+7n)$.
(17) $(xy-5z)(xy+2z)$.　(18) $(11-ax)(15+ax)$.　(19)
$(20-a)(21+a)$.　(20) $5x(a-13xy)(a+3xy)$.

EXERCISE XXXVII.—(Page 41).　(1) $(x+9)^2$.
(2) $a+13)^2$.　(3) $(m+17)^2$.　(4) $(y+1)^2$.　(5) $(z+10)^2$.
(6) $(x^2+7)^2$.　(7) $(x+6y)^2$.　(8) $(m+11n)^2$.　(9)
$(x^3+12)^2$.　(10) $(a-18)^2$.　(11) $(p-16q)^2$.　(12)
$(a-15b)^2$.　(13) $(2a-3b)^2$.　(14) $(a-2x)^2$.　(15) $(2ax$
$-7c)^2$.　(16) $(3am+5xy)^2$.　(17) $(4ax^2-b^2c^3)^2$.　(18)
$(4xy+3z)^2$.　(19) $(1-x^3)^2$.　(20) $(3a-2)^2$.　(21) $(a-bc)^2$.
(22) $(5-4n)^2$.　(23) $\{x(a^2+2y^2)\}^2$.　(24) $(7x^3+3yz^2)^2$.
(25) $(a^2b^2-1)^2$.　(26) $\{\sqrt{3}(ab-3)\}^2$.　(27) $(a+b+c)^2$.
(28) $(x+y-z)^2$.　(29) $(a-b+5)^2$.　(30) $(2m-3n-4p)^2$.

EXERCISE XXXVIII.—(Page 41).　(1) (x^2+x+1)
(x^2-x+1).　(2) $(a^2+a+1)(a^2-a+1)$.　(3) (x^2+2x+7)

$(x^2 - 2x + 7)$. (4) $(x^2 + 4x + 9)(x^2 - 4x + 9)$. (5) $(a^2 + 5a + 25)(a^2 - 5a + 25)$. (6) $(m^2 + 2m + 4)(m^2 - 2m + 4)$.
(7) $(a^2 + 4ab - b^2)(a^2 - 4ab - b^2)$. (8) $(x^2 + xy + y^2)$ $(x^2 - xy + y^2)$. (9) $(2a^2 + 2a - 1)(2a^2 - 2a - 1)$. (10)
$(9c^2 + 3cd + d^2)(9c^2 - 3cd + d^2)$. (11) $(x^2 + 2xy + 3y^2)$ $(x^2 - 2xy + 3y^2)$. (12) $(3x^2 + 2xy + 1)(3x^2 - 2xy + 1)$.
(13) $(a^2 + ay + y^2)(a^2 - ay + y^2)$. (14) $(x^2 + xy + 2y^2)$ $(x^2 - xy + 2y^2)$. (15) $(a^2 + 2ax + 3x^2)(a^2 - 2ax + 3x^2)$.
(16) $(a^2 + 2ax + 4x^2)(a^2 - 2ax + 4x^2)$. (17) $(x^2 + 3x + 5)$ $(x^2 - 3x + 5)$. (18) $(a^2 + 3a + 6)(a^2 - 3a + 6)$. (19)
$(2a^2 + 3ab + 4b^2)(2a^2 - 3ab + 4b^2)$. (20) $(x^2 + 2xy + 2y^2)$ $(x^2 - 2xy + 2y^2)$. (21) $(a^2 + 2abc + 2b^2c^2)(a^2 - 2abc + 2b^2c^2)$. (22) $(x^2 + xy + y^2)(x^2 - xy + y^2)(x^4 - x^2y^2 + y^4)$. (23) $(x^2 + x + 1)(x^2 - x + 1)(x^4 - x^2 + 1)$. (24)
$(2a + b)(2a - b)(a + 5b)(a - 5b)$. (25) $(2x + 3y)(2x - 3y)$ $(3x + 2y)(3x - 2y)$. (26) $(4m + 5n)(4m - 5n)(3m + 2n)$ $(3m - 2n)$. (27) $(2c^2 + 2c + 1)(2c^2 - 2c + 1)$. (28) $(p + q)$ $(p - q)(3p + q)(3p - q)$. (29) $(x^2 + 11x + 4)(x^2 - 11x + 4)$.
(30) $(3x^2 + 7xy - 4y^2)(3x^2 - 7xy - 4y^2)$. (31) $(a^2 + 3ac + 7c^2)(a^2 - 3ac + 7c^2)$. (32) $(x^2 + 3xy + 8y^2)(x^2 - 3xy + 8y^2)$. (33) $(m^2 + 5m + 9)(m^2 - 5m + 9)$. (34) $(x^2 + 3x + 1)$ $(x^2 - 3x + 1)$. (35) $(a^2 + 4a + 1)(a^2 - 4a + 1)$. (36)
$(x + 2y)(x + y)(x - 2y)(x - y)$.

EXERCISE XXXIX.—(Page 42.) (1) $(x + y)(x - y)$ $(x^2 - xy + y^2)(x^2 + xy + y^2)$. (2) $(a + b)(a - b)$ $(a^2 - ab + b^2)(a^2 + ab + b^2)$. (3) $(ab + 1)(ab - 1)(a^2b^2 - ab + 1)(a^2b^2 + ab + 1)$. (4) $(xy + z)(x^2y^2 - xyz + z^2)$.
(5) $(2x + a^2)(4x^2 - 2a^2x + a^4)$. (6) $(x^2 - 2a)(x^4 + 2ax^2 + 4a^2)$. (7) $(x^2 + y^2)(x^4 - x^2y^2 + y^4)$. (8) $(a^2 + b^2)$ $(a^4 - a^2b^2 + b^4)$. (9) $(3a - 4b)(9a^2 + 12ab + 16b^2)$. (10)
$(pq - 3r)(p^2q^2 + 3pqr + 9r^3)$. (11) $(x + 2y)(x^4 - 2x^3y + 4x^2y^2 - 8xy^3 + 16y^4)$. (12) $(x - y)(x^2 + xy + y^2)$ $(x^6 + x^3y^3 + y^6)$. (13) $(m + b)(m - b)(m^4 + m^3b + m^2b^2 + mb^3 + b^4)(m^4 - m^3b + m^2b^2 - mb^3 + b^4)$. (14)
$(x^{16} + y^{16})(x^8 + y^8)(x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$. (15) $(a - b)(a + b)(a^2 + b^2)(a^2 - ab + b^2)(a^2 + ab + b^2)$ $(a^4 - a^2b^2 + b^4)$. (16) $(2ab + 5c)(4a^2b^2 - 10abc + 25c^2)$.
(17) $(xyz - 1)(x^2y^2z^2 + xyz + 1)$. (18) $(a + b + c)(a^2 + 2ab$

$+b^2-ac-bc+c^2$). (19) $(x-y-z)(x^2-2xy+y^2+xz$ $-yz+z^2$). (20) $(a-b-1)(a^2-2ab+b^2+a-b+1)$. (21) $(1-x-y)(1+x+y+x^2+2xy+y^2)$. (22) $(a-b-c)$ $(a+b+c)(a^2+b^2+2bc-c^2)$. (23) $(a+b)(a^2-ab+b^2$ $+1)$. (24) $(x+y)^3$. (25) $(x-y)^3$. (26) $2c(c^2+3d^2)$. (27) $ab(3a+b)(3a-b)(9a^2+3ab+b^2)(9a^2-3ab+b^2)$. (28) $a^2(ax+2y)(ax-2y)(a^2x^2+2axy+4y^2)(a^2x^2-2axy$ $+4y^2)$. (29) $(a+b-c)(a^2+2ab+b^2+ac+bc+c^2)$. (30) $(x-y-z)(x^2-2xy+y^2+xz-yz+z^2)$.

EXERCISE XL.—(Page 42). A.—(1) $(3x+2)(x+1)$. (2) $(2a+1)(a+2)$. (3) $(3x+1)(x+3)$. (4) $(4a+1)$ $(a+2)$. (5) $(3x+5)(x+1)$. (6) $(x+2)(5x+4)$. (7) $(x+2)(2x+5)$. (8) $(a+2)(3a+4)$. (9) $(x+2)(7x+2)$. (10) $(a+5)(4a+3)$. (11) $(x+13)(3x+2)$. (12) $(4m+5)$ $(2m+7)$. (13) $(3x-5)(2x-7)$. (14) $(2x-5)(3x-4)$. (15) $(3x-2)(2x-1)$. (16) $(2x-7y)(4x-3y)$. (17) $(a-1)$ $(4a+9)$. (18) $(1-m)(7-3m)$. (19) $2(x-2)(2x-3)$. (20) $(x-7)(3x-2)$. (21) $(8x-5z)(7x-4z)$. (22) $(3b-c)$ $(8b-3c)$. (23) $(2x-3y)(12x-7y)$. (24) $(m-4n)$ $(56m-5n)$.

B.—(1) $(x+3)(4x-1)$. (2) $(a+6)(3a-5)$. (3) $(x+5)$ $(3x-1)$. (4) $(m+8)(2m-1)$. (5) $(c+3)(3c-2)$. (6) $(p+2)(4p-7)$. (7) $(4a+7)(3a-1)$. (8) $(1+7y)$ $(5-3y)$. (9) $(3x+7y)(4x-5y)$. (10) $13(a+b)(3a-2b)$. (11) $(2x-3)(3x+1)$. (12) $(a-3)(2a+5)$. (13) $(a-7)$ $(3a+2)$. (14) $(b-3)(12b+5)$. (15) $(3x+5)(4x-7)$. (16) $(x-3y)(2x+y)$. (17) $(3a-4b)(8a+b)$. (18) $(4-5c)(5+4c)$. (19) $(1-2n)(4+3n)$. (20) $(3x+y)$ $(13x-11y)$.

EXERCISE XLI.—(Page 43). (1) $(2x+3y+4z)$ $(3x+4y+5z)$. (2) $(2x+5y+3z)(3x+2y+2z)$. (3) $(4a+5b+6c)(3a+4b+7c)$. (4) $2(3a+2b+c)(3a+5b$ $+7c)$. (5) $(2x+5y+6)(9x+7y+6)$. (6) $(2a+3b+4)$ $(3a+5b+6)$. (7) $(x+3y-2z)(2x+y-3z)$. (8) $(4x-5y$ $+3z)(7x-2y-4z)$. (9) $(2a-b+5c)(5a-2b-3c)$. (10) $(7x-8y+3)(5x+2y-4)$. (11) $(9a-7b+4)$ $(3a+2b-11)$. (12) $(x+y+z)(x^2+y^2+z^2-xy-xz-yz)$.

(13) $(x+y-z)(x^2+y^2+z^2-xy+xz+yz)$.   (14) $(x-y+z)$
$(x^2+y^2+z^2+xy-xz+yz)$.   (15) $(x-y-z)(x^2+y^2+z^2$
$+xy+xz-yz)$.

EXERCISE XLII.—(Page 44). A.—(1) $x+y$. (2)
$x+y$. (3) a. (4) $2x-3y$. (5) $a^2b^2(a-b)$. (6) $x+y$.
(7) $x+2y$.   (8) $a(a-x)$.   (9) $b(a+b)$.   (10) $c-d$.
(11) $y(x-1)$.   (12) a^2+x^2.   (13) $x-5$.   (14) $x-10$.
(15) $x-12$.   (16) $x+3$.   (17) $x+2y$.   (18) $a+3$.
(19) $x+2$. (20) $x-3y$.   (21) $3x+1$. (22) $x-3$. (23)
$a+3b$. (24) $a(x+a)$.

B.—(1) $x^2-13x+5$.   (2) a^2-3a+2. (3) x^2-3x+7.
(4) $x-3$.   (5) $a+1$.   (6) $7x^2+3x-1$.   (7) $x-1$.
(8) $a-3$. (9) $3x-5a$. (10) None. (11) x^2+xy+y^2.
(12) x^2-2x+4.

EXERCISE XLIII.—(Page 45). A.—(1) $\dfrac{1}{ax-1}$.

(2) $2(x-y)$.   (3) $\dfrac{2a-c}{2a+c}$.   (4) $\dfrac{5x}{3y}$.   (5) $\dfrac{2(b+c)}{b^2}$.

(6) $\dfrac{x^2+x+1}{x^2-2x+1}$.   (7) $\dfrac{x^2-ax+a^2}{x^2-a^2}$.   (8) $\dfrac{b}{d}$.   (9) $\dfrac{x}{x^2-2z^2}$.

(10) $\dfrac{x}{x+1}$.   (11) $\dfrac{x}{2y}$.   (12) $\dfrac{3x+5}{2x+3}$.   (13) $\dfrac{a+b-c}{a+b+c}$.

(14) $\dfrac{4x^2+1}{5x^2+x+1}$.   (15) $\dfrac{2a+3b}{2a}$.   (16) $\dfrac{y}{bc}$.   (17) $\dfrac{3a}{a+2}$.

(18) $\dfrac{xy}{3az}$.   (19) $\dfrac{xy}{x-2}$.   (20) $\dfrac{1-x}{1+y}$.   (21) $\dfrac{x+3}{2}$.

(22) $\dfrac{x+1}{x+2}$.

B.—(1) $\dfrac{x+4}{x-2}$.   (2) $\dfrac{x+1}{x-3}$.   (3) $\dfrac{x^2-xy-y^2}{x^2+xy-y^2}$.

(4) $\dfrac{x-2}{x-3}$.   (5) $\dfrac{x+1}{x-3}$.   (6) $\dfrac{m+1}{m-1}$.   (7) $\dfrac{x^2+2x+4}{x^2+x-2}$.

(8) $\dfrac{x^2 - 3ax + 11a^2}{x^3 - 5a^2x + 7a^3}$. (9) $\dfrac{x^2 + 3x - 2}{x^3 - 2x^2 - 3}$. (10) $\dfrac{3x - 5}{x^2 - 5x + 6}$.

(11) $\dfrac{5x^2 - x - 3}{x^2 - 2x + 3}$. (12) $\dfrac{a - 2b}{a + 2b}$. (13) $\dfrac{a + 4}{a + 5}$.

(14) $\dfrac{2 - 2x + 4x^2}{3 - 3x + 9x^2}$. (15) $\dfrac{x^2 - 7ax + 12a^2}{2x^2 + 14ax + 24a^2}$.

(16) $\dfrac{x^2 - 2x + 3}{5x^2 - x - 3}$.

EXERCISE XLIV. —(Page 47). (1) $\dfrac{x - y}{x^2}$. (2) $\dfrac{4}{3}$.

(3) $\dfrac{4x + 3y}{x + 2}$. (4) $\dfrac{5a - b}{x(3a - 2)}$. (5) $\dfrac{x - 3}{x - 11}$. (6) $\dfrac{ab}{2a - 1}$.

(7) $\dfrac{x^2}{x^2 - 7x + 6}$. (8) $\dfrac{x - 3}{x - 6}$. (9) $\dfrac{x^2}{x^2 - 7x + 10}$. (10) $\dfrac{x + 1}{x + 4}$.

(11) $\dfrac{x}{x - 2}$. (12) $\dfrac{1}{(x + 4)^2}$. (13) $(x - 5)^2$. (14) 1.

(15) $\dfrac{(a - b)^5}{a^2}$. (16) $\dfrac{a - b + c}{a + b - c}$. (17) $\dfrac{x + y + z}{x - y - z}$.

(18) $\dfrac{x + m - n}{x - m + n}$. (19) x. (20) $\dfrac{x - 5}{x - 1}$. (21) $\dfrac{1}{(x - y)^2}$.

(22) $\dfrac{1}{(x + a)(x + b)(x - b)(x - c)}$. (23) $\dfrac{(x + y)^2}{x^2 + y^2}$.

(24) $\dfrac{2x - 1}{2x - 5}$. (25) $\dfrac{1}{a + b}$. (26) $x - 3$.

EXERCISE XLV·—(Page 48). (1) $x(x^2 - 1)$. (2) $ab(a + b)$. (3) $xy(4x^2 - 1)$. (4) $6x(3x - 1)$. (5) $a^3 + b^3$. (6) $ab(4a^2 - 1)$. (7) $x(x^2 - 4)$. (8) $(x + 1)(x - 1)^2$. (9) $(x + 2)^2(x + 3)$. (10) $(x - 1)(x - 2)(x - 4)$. (11) $(x - 5)(x - 6)(x + 7)$. (12) $(x + 3)(x + 4)(x + 5)$. (13) $(x - 2)(x + 2)(x - 11)$. (14) $(x + 1)(x - 2)(2x + 1)$. (15) $(x - 3)(x^2 + 3x + 9)(x - 12)$. (16) $(x + 4)^3(x^4 - 4x + 16)$. (17) $(x - 4)(x + 5)(x - 6)$. (18) $(x + 1)(x + 2)$

$(2x+1)$.   (19) $(x+2)(x+3)(5x+1)$.   (20) $(x-2)(x+2)$
$(3x-7)$.   (21) $6(x+2)(2x+1)(4x-7)$.   (22) $(x+y)$
$(2x-7y)(4x-5y)$.   (23) $(a+2b)(a-2b)(a^2-b^2)$ (24)
$(x+3)(x+4)(x^2+9x+20)$.

EXERCISE XLVI.—(Page 49).   A.—(1) $\dfrac{3x+5}{3}$.

(2) $\dfrac{4x+9}{5}$.   (3) $\dfrac{25x-16}{12}$.   (4) $\dfrac{17x}{36}$.   (5) $\dfrac{19x-201}{225}$.

(6) $\dfrac{12x^2+28x-17}{8x^2}$.   (7) $\dfrac{5x+29}{102x}$.   (8) $\dfrac{3a-8b}{8a}$.

(9) 0.   (10) $\dfrac{80x^3+64x^2+84x+45}{60x^2}$.   (11) $\dfrac{4y+3z}{yz}$.

(12) $\dfrac{a^3-b^3-c^3+abc}{abc}$.

(13) $\dfrac{5a^3c+2a^2bc+9ab-27ac+6c^2}{3a^2c^2}$.

(14) $\dfrac{11x^3-18x^2-27x-16}{30x^3}$.   (15) $\dfrac{6x-4a}{ax}$.   (16) $\dfrac{2x+5}{x^2+5x+6}$.

(17) $\dfrac{7x+31}{x^2+9x+20}$.   (18) $\dfrac{x-1}{x^2-9x+20}$.   (19) $\dfrac{2x+12}{x^2-4x-12}$.

(20) $\dfrac{(a+b)x-2ab}{x^2-(a+b)x+ab}$.   (21) $\dfrac{(a+b)x}{x^2+(a+b)x+ab}$.

(22) $\dfrac{4xy}{x^2-y^2}$.   (23) $\dfrac{2x^2}{x^2-y^2}$.   (24) $\dfrac{2}{x^2-8x+15}$.

(25) $\dfrac{20x}{x^2-25}$.   (26) $\dfrac{2x^3}{1-x^4}$.   (27) $\dfrac{2ab}{a^2-b^2}$.   (28) $\dfrac{2x^3}{x^2-y^2}$.

(29) 2b.   (30) $\dfrac{x^2+y^2}{xy(x^2-y^2)}$.

B.—(1) $\dfrac{2}{x+y}$.   (2) $\dfrac{2a}{a^2-x^2}$.   (3) $\dfrac{a^4+6a^2x^2+x^4}{a^4-x^4}$.

(4) $\dfrac{9x^2+34x+29}{(x+1)(x+2)(x+3)}$.   (5) $\dfrac{3x^2}{x^2-1}$.   (6) $\dfrac{4a^2+b^2}{4a^2-9b^2}$.

(7) $\dfrac{38x+14}{3(x^2-4)}$. (8) $\dfrac{11x-56}{(x-4)(x-5)(x-6)}$.

(9) $\dfrac{4x-18}{(x-2)(x-3)(x-4)}$. (10) 0. (11) 1. (12) $\dfrac{2y^2}{x^3-y^3}$.

(13) $\dfrac{7x+76}{(x+2)(x+3)(x+7)}$. (14) $\dfrac{x+c}{(x-a)(x-b)}$. (15) 0.

(16) 0. (17) 0. (18) 0. (19) $\dfrac{a^2+b^2}{2ab}$.

(20) $\dfrac{48a^3}{(x^2-a^2)(x^2-9a^2)}$.

EXERCISE XLVII.—(Page 51). (1) $7\frac{1}{7}$. (2) 15.
(3) 8. (4) 16. (5) 25. (6) 17. (7) 13. (8) $3\frac{7}{10}$.
(9) 4. (10) $-\frac{1}{4}$. (11) 60. (12) 8. (13) 45. (14) 7.
(15) 120. (16) 4. (17) 1. (18) 24. (19) 2. (20) 1.
(21) 5. (22) 41. (23) 17. (24) 3. (25) 3. (26) 2.
(27) 7. (28) 5. (29) $3\frac{1}{2}$. (30) 8.

EXERCISE XLVIII.— (Page 53). A.—(1) 72.
(2) 48. (3) 480. (4) 720. (5) 315. (6) 49 ; 50.
(7) 144 ; 128. (8) 98 ; 99 ; 100. (9) \$255 ; 204 ; 136.
(10) 436. (11) 18 ; 24. (12) \$724.80 ; 634.20. (13) 54.
(14) 100 ; 12. (15) \$346.20.

B.—(1) 35. (2) 78 ; 82 ; 160 ; 40. (3) 64. (4) 23.
(5) 36. (6) 54. (7) $4\frac{4}{7}$. (8) $6\frac{1}{2}$. (9) 25. (10) \$900 ;
750. (11) 39. (12) 12 ; 6.

EXERCISE XLIX.—(Page 56). A.—(1) 2 ; 3.
(2) 5 ; 6. (3) 6 ; 7. (4) 9 ; 7. (5) 12 ; 3. (6) 12 ; 4.
(7) 13 ; 3. (8) 13 ; 9. (9) 7 ; 17. (10) 4 ; 3.
B.—(1) 6 ; 12. (2) 12 ; 8. (3) 18 ; 12. (4) 14 ; 15.
(5) 3 ; 5. (6) 10 ; 5. (7) 7 ; 3. (8) 2 ; 3. (9) 5 ; 5.
(10) $\dfrac{2}{a+b}$; $\dfrac{2}{a-b}$.

EXERCISE L.—(Page 57). (1) 45 ; 40. (2) 48 ; 6.
(3) 72 ; 54. (4) 70 ; 32. (5) $\frac{11}{14}$. (6) 42 ; 38. (7) 94.
(8) \$10.80 ; \$4.50. (9) $4\frac{1}{2}$; $3\frac{3}{4}$. (10) 52 ; 48.

MISCELLANEOUS EXAMPLES. — (Page 58).
A.—(1) b. (2) $-8a-3b-c-6d$. (3) 4. (4) $4-12x$
$+25x^2-44x^3+46x^4-40x^5+25x^6$. (5) $-4x^2+10y^2$
$+5x^2$. (6) 11. (7) x^2+x-2 ; $x^5+2x^4+2x^3+2x^2+8$.

(8) $\dfrac{x}{a-x}$.   (10) $(10a-1)(a+8)$; $(3a-b)(3a+b)(9a^2$
$+3ab+b^2)(9a^2-3ab+b^2)$. (11) 13. (12) 1. (13)
$32x^5+76x^4-25x^3-198x^2-48x+135$. (14) $3-12x$
$+\dfrac{46}{5}x^2+8x^3$.   (15) $\dfrac{7bc}{13a^3}$.   (16) $\dfrac{3x^2+7x-12}{x^4-25x^2+144}$.

(17) $a-b$. (18) $(x-5y)(x+8y+1)$. (19) $(2a+3b)$
$(2a-3b)(x-2a)(x^2+2ax+4a^2)$; $(m-2)(m+2)(m^2$
$+2m+4)(m^2-2m+4)$. (20) 0. (21) $a^2b+b^2c+c^2a+ab^2$
$+bc^2+ca^2+2abc$. (22) $\dfrac{a^2b-a^2c+b^2c-ab^2+ac^2-bc^2}{(a-b)(b-c)(a-c)}$.

(23) 47. (24) 23. (25) 32. (27) x^6-64. (28) $x+6$.
(29) $15a^2+11a-12$. (30) $(a^3-1)x^2-a^2(a^2+2)x+a^3+1$.
(31) $16a^2b^2$. (32) $16ab^3$. (33) $(a-b)(a+b)(a^2+b^2)$
$(a^4+b^4)(a^8+b^8)$. (34) $\frac{1}{10}$. (35) 400. (36) 11025.

(37) $2x-9$. (38) $\dfrac{x-y}{4z}$. (39) 2. (40) $(a+b)(a-c)$. (41) x^2
$-2xy+y^2$. (42) $(x-26)(x-62)$. (43) $kl^2+lm-km$;
$\frac{3}{2}a^3-5a^2+\frac{1}{4}a+9$ (44) m. (45) $b+c-a$. (46)

$(2x^2-y^2)(3x-2y)$.   (47) $\dfrac{3x^2}{y}+\dfrac{2x}{y}-\dfrac{1}{y}+\dfrac{1}{xy}$.

(48) $-6a+3b+6c$. (49) $3m+10n+12p$. (50) $25a+8b$
$-3c$. (51) $147a-109b-167c$. (52) $2a^2b^2+2a^2c^2$
$+2b^2c^2-a^4-b^4-c^4$.   (53) $\dfrac{6ax-2by+bxy}{2axy}$; $5x^2$.

(54) $x^2 + 2y - y^2$.　(55) $(x-27)(x-33)$; $(a-2b+8)$ $(a+2b-7)$.　(56) 4.　(57) $7x^2 + 8x + 1$.　(58) 10. (60) 2.　(61) -1.　(62) $x-3$.　(63) 8.　(64) $(x-1)$

$(x-2)(x-3)$.　(65) $\dfrac{m^3 + 3mn^2}{4}$.　(66) $3a^4 + 10a^2b^2$

$+ 3b^4$.　(67) $4x^8 + 7x^6 + 6x^4 + 7x^2 + 4$.　(68) $8a^3b$. (69) $16x^4 - 32x^3 - 520x^2 - 536x + 3465$.　(70) 100. (71) 6.　(72) -34.　(73) $x^6 + 4x^5 + 2x^4 + 9x^2$ $-4x+4$.　(74) $2ab + 3ac + 6bc$.　(75) $-4a^2b^2$. (76) $12a^4 - 17a^3b - 9a^2b^2 + 13ab^3 - 63b^4$.　(77) $4a^2 + 8ab$ $+ 16b^2$.　(78) $m^4 + 2m^3 + 3m^2 + 2m + 1$.　(79) -126. (80) 11.　(81) $2yz - y^2$.　(82) abc; xyz.　(83) $x+1$.

(84) 1.　(85) $42(a^4 + a^3b - ab^3 - b^4)$.　(86) $\dfrac{4x}{x+1}$.

(87) $\dfrac{5}{(x-2)(x-3)(5-x)}$.　(88) 5.　(89) 5.　(90) $x^3 + 6x^2$

$+ 11x + 6$.　(91) $21a - 27b + 6c$.　(92) $d(d-2c)$. (93) $x^6 - 2a^3x^3 + a^6$.　(94) $1 + x - x^3 - x^4$.　(95) $a + 3b$ $+ 4x + 4y$.　(96) $x^4 - a^4$.　(97) $3a^3 - 8a^2b - 4ab^2 + 3b^3$. (98) 5.　(99) $3\frac{1}{7}$.　(100) 101.

B.—(1) 63.　(2) 24.　(3) 72 lbs.　(4) 5 days. (5) 60 gal.　(6) $2.43\frac{7}{11}$.　(7) $6\frac{2}{3}$; $13\frac{1}{3}$.　(8) 6.30 p.m. (9) 96.　(10) 70.　(11) 2560.　(12) \$1600.　(13) 24. (14) 5 miles.　(15) 18 and 12.　(16) 12 and 6. (17) \$1250.　(18) 2; $10\frac{1}{2}$; $2\frac{1}{2}$ lbs.　(19) £240.　(20) 22. (21) 2450; 196; 98.　(22) 69; 81.　(23) 84.　(24) 15 by 11 feet.　(25) 1540; 880; 616.　(26) 110 yds.　(27) 30 hrs.　(28) 24; 30; 9; 81.　(29) 2 ft.　(30) 8 pence. (31) 751; 752.　(32) 16.　(33) 24 ft.　(34) 88.　(35) \$346.20.　(36) 36; 54; 72.　(37) \$719; 791; 971. (38) \$246; 369; 492.　(39) 39.　(40) 72; 60.　(41) \$135; \$375; \$235.　(42) \$75; 150; 225; 300.　(43) \$2345; \$3456; \$4567; \$7891.　(44) 14; 15.　(45) 25. (46) 42c.; 7c.　(47) 74c.; 29c.　(48) 30; 50 miles an hour.　(49) 5; 7.　(50) \$2.25; \$1.35.

C.—(1) $12x^4 + 38x^3 + 58x^2 + 78x + 54$. (2) $20x^4 + 58x^3 + 82x^2 + 106x + 70$. (3) $30a^4 - 4a^3 - 6a^2 - 8a - 64$. (4) $42a^4 - 5a^3 - 7a^2 - 9a - 81$. (5) $16x^4 - 52x^3 + 10x^2 - 42x - 12$. (6) $54x^4 - 69x^3 + 56x^2 - 79x + 28$. (7) $27x^5 + 48x^4 + 82x^3 + 66x^2 + 59x + 30$. (8) $48a^5 - 124a^4 + 138a^3 - 122a^2 + 87a - 27$. (9) $55a^5 + 111a^4 - 58a^3 - 130a^2 + 19a + 35$. (10) $24x^4 + 71x^3y + 193x^2y^2 + 109xy^3 + 38y^4$. (11) $15a^4 - 56a^3b + 99a^2b^2 - 100ab^3 + 42b^4$. (12) $88x^4 + 15x^3y + 32x^2y^2 - 95xy^3 - 88y^4$. (13) $36x^6 + 85x^5 + 148x^4 + 178x^3 + 208x^2 + 157x + 88$. (14) $16x^6 - 42x^5 + 79x^4 - 106x^3 + 141x^2 - 113x + 70$. (15) $14a^6 - 40a^5b + 76a^4b^2 - 64a^3b^3 + 52a^2b^4 - 36ab^5 + 18b^6$. (16) $24x^3 + 98x^2 + 133x + 60$. (17) $210x^3 - 737x^2 + 862x - 336$. (18) $105x^3 + 386x^2y + 472xy^2 + 192y^3$. (19) $192a^3 - 680a^2b + 802ab^2 - 315b^3$. (20) $336x^6 + 1154x^5 + 2621x^4 + 3480x^3 + 3379x^2 + 1918x + 720$. (21) $6a^6 - 29a^5b + 86a^4b^2 - 150a^3b^3 + 184a^2b^4 - 133ab^5 + 60b^6$. (22) $162x^6 - 387x^5y + 594x^4y^2 - 530x^3y^3 + 336x^2y^4 - 123xy^5 + 28y^6$. (23) $308x^6 - 775x^5 + 1120x^4 - 1174x^3 + 852x^2 - 439x + 120$. (24) $36x^6 - 25x^4 - 26x^2 - 225$. (25) $144a^6 - 121a^4 - 334a^2 - 1225$. (26) $a^8 - x^8$. (27) $x^4 - 10x^2 + 9$. (28) $x^8 + x^4 + 1$. (29) $x^6 - 22x^4 + 60x^3 - 55x^2 + 12x + 4$. (30) $a^5 - a^4b - 2a^3b^2 + 2a^2b^3 + ab^4 - b^5$. (31) $x^3 + y^3 + 3xy - 1$. (32) $a^3 + b^3 - c^3 + 3abc$. (33) $60x^4 + 42x^3a - 107x^2a^2 + 10xa^3 + 14a^4$. (34) $12x^6 - 56x^5 + 159x^4 - 269x^3 + 318x^2 - 224x + 96$. (35) $729a^8 + 1296a^7 - 4176a^6 - 1632a^5 + 8590a^4 - 9744a^3 + 7056a^2 - 2592a + 729$.

D.—(1) $7x^2 + 4x + 5$ and 10. (2) $8x^2 + 5x + 7$ and 5. (3) $7x^2 + 9x + 8$ and 9. (4) $4a^2 + 12a + 11$ and 15. (5) $5x^2 + 6x + 7$ and 5. (6) $2a^2 - 8a - 5$ and 17. (7) $4x^2 + 7xy - 8y^2$ and 19. (8) $9x^2 - 2xy + 6y^2$ and 23. (9) $8a^2 - ab + 7b^2$ and 6. (10) $12a^2 - 5ab - 9b^2$ and 18. (11) $3x^2 + 4x + 5$ and $2x + 6$. (12) $5x^2 + 7x + 9$ and $x + 5$. (13) $2x^2 + x + 8$ and $2x + 17$. (14) $7x^2 - 2x - 9$ and $3x - 9$. (15) $3x^2 - 4x - 2$ and $5x - 7$. (16) $2x^2 - xy - 7y^2$ and $2xy^3 + 8y^4$. (17) $3x^2 - 5xy + 6y^2$ and $2xy^3 + 3y^4$. (18) $3x^3 + 4x^2 + 5x + 6$ and 5. (19) $2x^3 - 3x^2 + 4x - 5$

and 16. (20) $5x^3 - 4x^2 + 3x - 2$ and -7. (21) $a^2 + b^2 + c^2 - ab - ac - bc$. (22) $a^2 + b^2 + c^2 - ab + ac + bc$. (23) $a^2 + b^2 + c^2 + ab - ac + bc$. (24) $a^2 + b^2 + c^2 + ab + ac - bc$. (25) $x^2 + y^2 + 4 - xy - 2x - 2y$. (26) $x^2 + y^2 + 9 - xy + 3x + 3y$. (27) $x^2 + y^2 + 1 + xy - x + y$. (28) $4x^2 + 9y^2 + 16z^2 - 6xy - 8xz - 12yz$ (29) $9x^2 + y^2 + 4z^2 + 3xy - 6xz + 2yz$. (30) $25x^2 + 9y^2 + 36 - 15xy + 30x + 18y$. (31) $x^4 - 8x^2 + 16$. (32) $x^2 + x - 2$. (33) $2x^3 - 8x^2 + 3x - 12$. (34) $a + x$. (35) $x^2 - a^2$.

E.—(1) $\dfrac{26x^2 + 38xy}{35x^2 + 66xy + 27y^2}$. (2) $\dfrac{15x^2 - 4xy + 2y^2}{(3x + 4y)(4x + 5y)}$.

(3) 2. (4) $\dfrac{x^8 - x^4 + 1}{x^8 + x^4 + 1}$. (5) $\dfrac{16a^{15}}{1 - a^{16}}$. (6) $\dfrac{1}{2}$.

(7) $\dfrac{2x}{n}$. (8) $\dfrac{6x - 9}{(x^2 - 1)(2x + 3)}$. (9) $\dfrac{1}{a + 2}$. (10) $\dfrac{2xy^2}{x^4 - y^4}$.

(11) $\dfrac{4x}{x + a}$. (12) $\dfrac{51x - 4y}{84}$. (13) $\dfrac{3x^2 + 29x + 58}{(x + 2)(x + 3)(x + 4)}$.

(14) $\dfrac{x^2 + 4x - 65}{(x - 3)(x - 5)(x - 8)}$. (15) $\dfrac{x^2 + x - 168}{(x - 3)(x + 7)(x + 8)}$.

(16) $\dfrac{3a}{a + b}$. (17) $\dfrac{y}{x + y}$. (18) $\dfrac{1}{a - b}$. (19) $\dfrac{4x}{1 + 4x^2}$. (20) 1.

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